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On Similarity Measures of Fuzzy Soft Sets

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Abstract

In this paper several similarity measures of fuzzy soft sets are introduced. The measures are examined based on the geometric model, the set-theoretic approach and the matching function. A comparative study of these measures is done.

Keywords: *Fuzzy soft set, Matching function, Proximity measure, Similarity measure.*

1 Introduction

Uncertainty is present in almost every sphere of our daily life. Traditional mathematical tools are not sufficient to handle all the practical problems in fields such as medical science, social science, engineering, economics etc involving uncertainty of various types. Zadeh [21], in 1965, was the first to come up with his remarkable theory of fuzzy set for dealing these types of uncertainties where conventional tools fail. His theory brought a grand paradigmatic change in mathematics. Later there are theories namely the theory of intuitionistic fuzzy sets, vague sets, rough sets, interval mathematics etc to name a few, all are intended to become a tool for handling the uncertainty. All these theories are successful to some extent in dealing with the problems arising due to the vagueness present in the real world. But there are also cases where these theories failed to give satisfactory results, possibly due to the inadequacy of the parameterization tool in them.
Then in 1999, Molodtsov [16] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainty. Possible applications of soft set in various problems such as smoothness of functions, game theory, operation research, Rieman integration, Perron integration, probability theory, measurement theory, economics, medical science etc are shown by Molodtsov [16] and others [2, 12, 13]. H. aktas and N. Cagman [1] has shown that every fuzzy set and every rough set can be considered as a soft set. In that sense we can say that this theory is much more general than its predecessors.

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Also in several problems it is often needed to compare two sets. The sets may be fuzzy, may be vague etc. We often interested to know whether two patterns or images are identical or approximately identical or at least to what degree they are identical. Several researchers like Chen [3, 4, 5], Li and Xu [10], Hong and Kim [7], C.P. Pappis [18, 19] and many others [6, 8, 9] has studied the problem of similarity measurement between fuzzy sets, fuzzy numbers and vague sets. Recently P. Majumdar and S. K. Samanta [14, 15] have studied the similarity measure of soft sets and intuitionistic fuzzy soft sets. Similarity measures have extensive application in several areas such as pattern recognition, image processing, region extraction, Psychology [17], handwriting recognition [11], decision making [20], coding theory etc.
The main purpose of this paper is to introduce the concept of similarity between fuzzy soft sets. And that has been done using three different approaches. A comparative study was also done at the end.

2 Preliminaries

In this section we briefly review some definitions and examples which will be used in rest of the paper.

Definition 2.1[12] Let U be an initial universal set and let E be a set of parameters. Let $I^U, (I = [0,1])$ denote the power set of all fuzzy subsets of U. Let $A \subset E$.
A pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by $F : A \to I^U$.

Example 2.2 As an illustration, consider the following example.
Suppose a soft set (F, E) describes attractiveness of the shirts which the authors are going to wear. U = the set of all shirts under consideration = $\{x_1, x_2, x_3, x_4, x_5\}$ Let I^U be the collection of all fuzzy subsets of U. Also let E = {colorful, bright, cheap, warm} $= \{e_1, e_2, e_3, e_4\}$.

Let $F(e_1) = \{\frac{x_1}{0.5}, \frac{x_2}{0.9}, \frac{x_3}{0}, \frac{x_4}{0}, \frac{x_5}{0}\}$ $F(e_2) = \{\frac{x_1}{1.0}, \frac{x_2}{0.8}, \frac{x_3}{0.7}, \frac{x_4}{0}, \frac{x_5}{0}\}$, $F(e_3) = \{\frac{x_1}{0}, \frac{x_2}{0}, \frac{x_3}{0}, \frac{x_4}{0.6}, \frac{x_5}{0}\}$,
$F(e_4) = \{\frac{x_1}{0}, \frac{x_2}{1.0}, \frac{x_3}{0}, \frac{x_4}{0}, \frac{x_5}{0.3}\}$.
So, the fuzzy soft set (F, E) is a family $\{F(e_i), i = 1,2,3,4\}$ of I^U .

Definition 2.3[12] For two fuzzy soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a fuzzy soft subset of (G, B) if $(i) A \subset B$, $(ii) \forall \varepsilon \in A, F(\varepsilon)$ is a fuzzy subset of $G(\varepsilon)$.

Definition 2.4[12] (Equality of two fuzzy soft sets) Two soft sets (F, A) and (G, B) over a common universe U are said to be fuzzy soft equal if (F, A) is a fuzzy soft subset of (G, B) and (G, B) is a fuzzy soft subset of (F, A).

Definition 2.5[12] (Null fuzzy soft set) A soft set (F, A) over U is said to be null fuzzy soft set denoted by Φ, if $\forall \varepsilon \in A, F(\varepsilon) =$ null fuzzy set of U .

* Corresponding author

Definition 2.6[12] (Absolute fuzzy soft set) A soft set (F, A) over U is said to be absolute fuzzy soft set denoted by $\tilde{A}$, if $\forall \varepsilon \in A, F(\varepsilon) = U$.

Definition 2.7[12] Union of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B,$ and $\forall e \in C,$
$H(e) = F(e),\ e \in A - B,\ = G(e), e \in B - A,\ = F(e) \cup G(e), e \in A \cap B.$
We denote the union as $(F, A) \bigcup (G, B)$.

Definition 2.8[14] Intersection of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B,$ and $\forall e \in C,\ H(e) = F(e) \cap G(e).$ We denote the intersection as $(F, A) \bigcap (G, B)$.

3 Similarity measure of two fuzzy soft sets based on matching function

In this paper we redefine a fuzzy soft set for greater computational facilities. We also assume that the universal set U and the parameter set E are finite. Then we define a fuzzy soft set as follows:

Definition 3.1 Let U be an initial universal set and let E be a set of parameters. Let I^U denote the collection of all fuzzy subsets of U. A pair (F, E) is called a fuzzy soft set over U, where F is a mapping given by $F : E \rightarrow I^U$.

Actually definitions 1 and 8 are the same because if we take any proper subset A of E and assign the e- approximation $F(e) = \underline{0}\ \ \forall e \in E \setminus A,$ then the fuzzy soft set (F, A) and (F, E) bear the same meaning.
Let U be the universe and E, the set of parameters. Then we can express a fuzzy soft set over U as a matrix. We illustrate the process with an example. Consider the example 2. Then the $(i, j)^{th}$ entry of the matrix is the membership value of $F(e_i)(x_j)$ if $e_i \in A,$ and it is equal to 0 if $e_i \notin A,$ then we get a matrix called a fuzzy membership matrix as below:

* Corresponding author

Let $\hat{A}=\begin{pmatrix}0.5 & 1.0 & 0 & 0\\ 0.9 & 0.8 & 0 & 1.0\\ 0 & 0.7 & 0 & 0\\ 0 & 0 & 0.6 & 0\\ 0 & 0 & 0 & 0.3\end{pmatrix}$. Then with the above interpretation the fuzzy soft set (F,A) is represented by the matrix A and we write $(F,A)=\hat{A}$. Clearly, the complement of (F,A), i.e. $(F,A)^C$ will be represented by another matrix $\hat{B}$ where $\hat{B}=\begin{pmatrix}0.5 & 0 & 1.0 & 1.0\\ 0.1 & 0.2 & 1.0 & 0\\ 1.0 & 0.3 & 1.0 & 1.0\\ 1.0 & 1.0 & 0.4 & 1.0\\ 1.0 & 1.0 & 1.0 & 0.7\end{pmatrix}$.

Henceforth we will denote a column of the fuzzy membership matrix by the vector $\overrightarrow{F}(e_i)$, or by simply $F(e_i)$, e.g. here $\overrightarrow{F}(e_1)=(0.5,0.9,0,0,0)$ in $\hat{A}$.

Next we define similarity measure using a matching function.

Definition 3.2 Let (F,E) and (G,E) be two fuzzy soft sets over U. Then the similarity between them, denoted by $S(F,G)$ or $S_{F,G}$, is defined by $S(F,G)=S_{F,G}=\dfrac{\sum_{i=1}^{n}\{\vec{F}(e_i)\bullet\vec{G}(e_i)\}}{\sum_{i=1}^{n}\{(\vec{F}(e_i))^2\vee(\vec{G}(e_i))^2\}}$.

The following is an example to illustrate the above definition.

Example 3.3 Let (F,E) and (G,E) be two fuzzy soft sets over U having the fuzzy membership matrix as follows:

$$\hat{A}=\begin{pmatrix}0.2 & 0.5 & 0 & 0\\ 0.7 & 0.3 & 0 & 0.7\\ 0 & 1.0 & 0 & 0\\ 0 & 0 & 0.95 & 0\\ 0 & 0 & 0 & 0.2\end{pmatrix} \text{ and } \quad \hat{B}=\begin{pmatrix}0.13 & 0.4 & 0 & 0\\ 0.6 & 0.1 & 0.1 & 0.4\\ 0.1 & 0.3 & 0.3 & 0\\ 0 & 0 & 0.8 & 0\\ 0 & 0 & 0 & 0.1\end{pmatrix},$$

where $U=\{x_1,x_2,x_3,x_4,x_5\}$ and the set of parameters is $E=\{e_1,e_2,e_3,e_4\}$. Then

$$S(F,G)=S_{F,G}=\frac{\sum_{i=1}^{4}\{\vec{F}(e_i)\bullet\vec{G}(e_i)\}}{\sum_{i=1}^{4}\{(\vec{F}(e_i))^2\vee(\vec{G}(e_i))^2\}}\cong 0.617.$$

* Corresponding author

Proposition 3.4 *Let* (F,E) *and* (G,E) *be two fuzzy soft sets over* U *. Then the following holds:*
(i) $S_{F,G} = S_{G,F}$, *(ii)* $(F,E) = (G,E) \Rightarrow S_{F,G} = 1$, *(iii)* $(F,E) \bigcap (G,E) = \Phi \Leftrightarrow S_{F,G} = 0$
and (iv) if $(F,E) \subseteq (H,E) \subseteq (G,E),$ *then* $S_{F,G} \leq S_{H,G}$

Proof. Trivial

4 Similarity measure based on set theoretic approach

Let $U = \{x_1, x_2, ..., x_n\}$ be the universal set of elements and $E = \{e_1, e_2, ..., e_m\}$ be the universal set of parameters. Let $\hat{F} = (F,E)$ and $\hat{G} = (G,E)$ be two fuzzy soft sets over (U,E). Then $\hat{F} = \{F(e_i) \in P(U); e_i \in E\}$, $\hat{G} = \{G(e_i) \in P(U); e_i \in E\}$, where $F(e_i)$ is called the e_i-th approximation of $\hat{F}$ and $G(e_i)$ is called the e_i-th approximation of $\hat{G}$.
$P(U)$ be the collection of all fuzzy subsets of U.
Let $M(\hat{F}, \hat{G})$ indicates the similarity between the soft sets $\hat{F}$ and $\hat{G}$. To find the similarity between $\hat{F}$ and $\hat{G}$, first we have to find the similarity between their e- approximations. Let $M_i(\hat{F}, \hat{G})$ denote the similarity between the two e_i approximations $F(e_i)$ and $G(e_i)$.

Definition 4.1 Let us define $M_i(\hat{F}, \hat{G}) = \dfrac{\sum_{j=1}^{n} (F_{ij} \wedge G_{ij})}{\sum_{j=1}^{n} (F_{ij} \vee G_{ij})}$, where $F_{ij} = F(e_i)(x_j) \in I$ and $G_{ij} = G(e_i)(x_j) \in I$. Then $M_{F,G} = M(\hat{F}, \hat{G}) = \max_i M_i(\hat{F}, \hat{G})$.

An example is given to illustrate the above definition.

Example 4.2 Consider the following two fuzzy soft sets where $U = \{x_1, x_2, x_3, x_4\}$ and $E = \{e_1, e_2, e_3, e_4\}$:

$$\hat{F} = \begin{pmatrix} 0.2 & 0.5 & 0.9 & 1.0 \\ 0.1 & 0.2 & 0.6 & 0.5 \\ 0.5 & 0.4 & 0.3 & 0.2 \\ 0.1 & 1.0 & 0.3 & 0.4 \end{pmatrix} \text{ and } \hat{G} = \begin{pmatrix} 0.4 & 0.3 & 0.2 & 0.9 \\ 0.6 & 0.5 & 0.2 & 0.1 \\ 0.4 & 0.3 & 0.2 & 0.1 \\ 1.0 & 0.9 & 0.8 & 0.7 \end{pmatrix}.$$

Then $M_1 = \dfrac{0.8}{2.5} = 0.32$, $M_2 = 0.71$, $M_3 = 0.35$, $M_4 = 0.63$
Hence $M_{F,G} = \max\{M_1, M_2, M_{3,} M_4\} = M_2 = 0.71$

* Corresponding author

Proposition 4.3 *Let* $\hat{F} = (F, E)$ *and* $\hat{G} = (G, E)$ *be two fuzzy soft sets over* (U, E). *Then the following conditions hold:*

(i) $M_{F,G} = M_{G,F}$, *(ii)* $\hat{F} = \hat{G} \Rightarrow M_{F,G} = 1$, *(iii)* $\hat{F} \cap \hat{G} = \Phi \Leftrightarrow M_{F,G} = 0$ *and*

(iv) $\hat{F} \subset \hat{H} \subset \hat{G} \Rightarrow M_{F,G} \le M_{H,G}$

Proof. Can be easily proved from the definitions.

Note 4.4 Also here $M_{F,G} = 1$ does not imply $\hat{F} = \hat{G}$.

5 Similarity measure based on distance

We know that if A and B are two fuzzy sets and the distance between them is d, then the similarity between them can be defined as $S = \frac{1}{1+d}$. Again a fuzzy soft set is a collection of its $e-approximations$ which are nothing but fuzzy sets. Here we take the distance between A and B as $d_\infty(A, B) = \max_i |a_i - b_i|$, where $A = (a_1, a_2, \ldots\ldots, a_n)$ and $B = (b_1, b_2, \ldots\ldots, b_n)$ are the two fuzzy sets. Then the similarity between them will be $T(A, B) = \frac{1}{1 + d_\infty(A, B)}$.

Now let $(F, E) = \{F(e_i), i = 1,2.....,n\}$ and $(G, E) = \{G(e_j), j = 1,2,....,n\}$ be two fuzzy soft sets where $F(e_i)$ is the e_i-th approximation of (F, E) and $G(e_i)$ is the e_i-th approximation of (G, E). Let $T_i(F, G)$ denotes the similarity between the $e-approximations$ $F(e_i)$ and $G(e_i)$.

So $T_i(F, G) = \frac{1}{1 + d_\infty^i}$, where d_∞^i is the distance between the $e-approximations$ $F(e_i)$ and $G(e_i)$.

Then the similarity measure between (F, E) and (G, E) will be denoted by $T(F, G)$ and is defined by

$$T(F, G) = \min_i T_i(F, G)$$

Example 5.1 Consider the following two fuzzy soft sets, where $U = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2, e_3\}$:

$$F = \begin{pmatrix} 0.2 & 0.9 & 1.0 \\ 1.0 & 0.1 & 0.5 \\ 0.4 & 0.2 & 0.4 \end{pmatrix} \text{ and } G = \begin{pmatrix} 0.6 & 0.9 & 0.1 \\ 0.7 & 1.0 & 0.5 \\ 0.1 & 1.0 & 0.4 \end{pmatrix}$$

Then, $d_\infty^1 = 0.4$, $d_\infty^2 = 0.9$ and $d_\infty^3 = 0.9$. Hence $T_1 = \frac{1}{1+0.4} = 0.71$, $T_2 = \frac{1}{1+0.9} = 0.53$ and $T_3 = \frac{1}{1+0.9} = 0.53$. $\therefore T_{F,G} = \min_i T_i = 0.53$

* Corresponding author

Proposition 5.2 *Let* (F, E) *and* (G, E) *be two fuzzy soft sets over* (U, E). *Then the following holds:*

(i) $T_{F,G} = T_{G,F}$, *(ii)* $(F, E) = (G, E) \Leftrightarrow T_{F,G} = 1$,

(iii) $\hat{F} \subset \hat{H} \subset \hat{G} \Rightarrow T_{F,G} \leq T_{H,G}$, *for any soft set* (H, E) *over* (U, E)

Note 5.3 The following property does not hold here:

(i) $\hat{F} \cap \hat{G} = \Phi \Leftrightarrow T_{F,G} = 0$

6 A comparative study

In this paper we have discussed three types of similarity measure of fuzzy soft sets based on the matching function $S_{F,G}$, the set-theoretic approach $M_{F,G}$ and the geometric model $T_{F,G}$. Some of the properties are common to all of them but few properties are exclusive for particular measures. The table 1 given below gives a comparison between the three measures. From this table we can have an idea about suitability of a particular measure for a particular application.

Table 1: Comparison table of three types of measures of similarity

Property	**S**	**M**	**T**
$X_{F,G} = X_{G,F}$	*Y*	*Y*	*Y*
$(F, E) = (G, E) \Leftrightarrow X_{F,G} = 1$	*Y*	*N*	*Y*
$(F, E) \cap (G, E) = \Phi \Leftrightarrow X_{F,G} = 0$	*N*	*Y*	*N*
$(F, E) \subseteq (H, E) \subseteq (G, E) \Rightarrow X_{F,G} \leq X_{H,G}$	*Y*	*Y*	*Y*

Here in the table1, S denotes the similarity measure based on a matching function, M denotes the measure based on set theoretic approach and T denotes the measure based on geometric model.

7 Conclusion

This paper introduces the notion of similarity between two fuzzy soft sets. We have introduced three measures of similarity for comparing two fuzzy soft sets. We have studied few properties of these three measures and at the end compared the properties of all of them. The similarity measures have natural applications in the field of pattern recognition, feature extraction, region extraction, image processing, coding theory etc.

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