

Multiobjective Optimization Approaches in Image Segmentation – The Directions and Challenges

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Abstract

A new trend of problem formulation for image segmentation is to use multiobjective optimization approach in its decision making process. Multiobjective formulations are realistic models for many complex optimization problems. In many real-life problems, objectives under consideration conflict with each other, and optimizing a particular solution with respect to a single objective can result in unacceptable results with respect to the other objectives. The aim of this paper is to provide a comprehensive review of multiobjective optimization in image segmentation problem. The models are classified according to relevant features, such as the various aspects of the optimization approaches used, various possible problem formulations, type of datasets modelled and the scope of the applications. Through our analysis of the current state of the research, we diagnose some of the directions and challenges for modelling the image segmentation problem with multiple objectives criteria. In this review, we consolidate the selected material in the literature, including more than 80 studies dated 1990 or later.

Keywords: *Multiobjective Optimization, soft-computing techniques, image segmentation, clustering, classification*

1 Introduction

Image segmentation is a critical and essential component of image analysis system. It is one of the most difficult tasks in image processing because it determines the quality of the final result of analysis [1]. Segmentation is basically clustering of the pixels in the image according to some criteria. The aim is to recognize homogeneous regions within an image as distinct and belonging to different objects. Typical pattern clustering activity involves a sequence of steps [2] where there is a decision to make from a list of available options at each step. One has to consider multiple perspectives in terms of its specific goal of segmentation so the desired output can be achieved. For instance, one may need to consider the type of pattern representation (optionally including feature extraction and/or selection), and the definition of pattern proximity measure, appropriate to the data domain, related to multiple criteria.

A new trend of problem formulation for image segmentation is to use approaches with multiple objectives in its decision making process [3-8]. For problems with multiple objectives, the objective functions defined are generally conflicting, preventing simultaneous optimization of each objective. Real world image segmentation problems actually do have multiple objectives, i.e., minimize overall deviation (intra-cluster spread of data), maximize connectivity (inter-cluster connectivity), minimize the number of features or minimize the error rate of the classifier etc. Consideration of these objectives combination is a difficult problem, causing a gap between the natures of image segmentation problem with real-world solution. A multiobjective optimization approach is an appropriate method to bridge this gap [9-11]. Although the studies on the use of approaches with multiple objectives have grown to a significant amount since 1995, but an extensive review has yet appeared.

The purpose of this study is to provide a recent literature review in this arena. First, we explain how image segmentation is a problem of multiple objectives. In this section, we identify multiple objectives associated with image segmentation problems. To achieve a simple and rough classification scheme, in Section 3, we describe type of datasets used in current applications and the current image segmentation techniques like clustering and classification with multiobjective optimization method. In Section 4, we give a detail study of the use of multiobjective optimization methods with classification approaches. We then focus on the clustering methods with multiple objectives in Section 5. Other less popular uses of multiobjective optimization solution are discussed in Section 6. Last but not least, Section 7 summarizes the design issue before we conclude the study.

2 Image Segmentation is a multiple objectives problem

Typical image segmentation activities [2] involves several processes. First, *pattern representation* refers to the number of classes, the number of available patterns, and the number, type, and scale of the features available to the clustering algorithm. Some of this information may not be controllable by the practitioner. After a pattern representation has been selected, another important activity is feature selection and extraction. *Feature selection* is the process of identifying the most effective subset of the original features to use in clustering. Meanwhile, *feature extraction* is the use of one or more transformations of the input features to produce new salient features. Either or both of these techniques can be used to obtain an appropriate set of features to use in clustering. In the context of selecting the list of suitable features, there is a possibility of selecting a list of multiple features based on the goal of the image segmentation. For instance, in segmenting a medical image based on CT scan, multiple features related to intensity, shape and spatial relationship can be considered.

After feature selection and extraction process, the following activity is pattern proximity. *Pattern proximity* is usually measured by a distance function defined on pairs of patterns. A simple distance measure like Euclidean distance can often be used to reflect dissimilarity between two patterns, whereas other similarity measures can be used to characterize the conceptual similarity between patterns. Consideration of the interpattern similarity is an issue related to multiple criteria. For example, one may have to consider spatial coherence with feature homogeneity when segmenting a medical image. Other possible criteria include inter-region connectedness versus intra-region compactness. The grouping output could affect subsequent feature extraction and similarity computations. The *grouping* step can be performed in a number of ways. The output result (Figure 1) can be represented in hard or fuzzy representation. In Figure 1(a), the boundary of the lake and forest is represented with hard/crisp representation where the partition of the data is divided clearly into two groups. In Figure 1(b), when the boundary of lake and forest is represented with fuzzy information, each pattern has a variable degree of membership in each of the output clusters. Similarly, algorithms produce variety of partitions/groups based on a set of selected criteria. For example, hierarchical clustering algorithms produce a nested series of partitions based on a criterion for merging or splitting clusters based on similarity. Partitional clustering algorithms identify the partition that optimizes (usually locally) a clustering criterion.

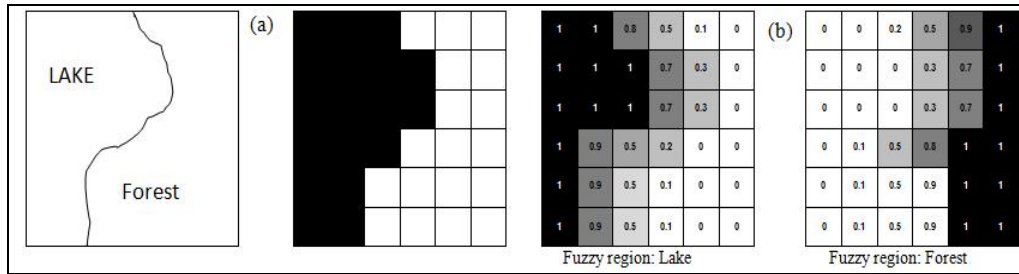


Fig. 1 Crisp versus fuzzy region (a) crisp representation (b) fuzzy representation

Finally, *cluster validity* analysis is the assessment of a clustering procedure's output. Often this analysis uses a specific criterion of optimality; however, these criteria are usually arrived at subjectively. Hence, little in the way of 'gold standards' exist in clustering except in well-prescribed subdomains. Validity assessments are objective and are performed to determine whether the output is meaningful. In deciding the type of cluster validity measurement, multiple objectives can be formulated with multiple validity indexes. Optimization of these indexes aims to achieve an optimal number of clusters in image segmentation problem.

The consideration of multiple criteria (objectives) starts from the understanding the data point of view to its selected segmentation process involved and finally to its assessment of its output. As visualized in Figure 2, there are possibly multiple sources of information for a specific segmentation problem, thus multiple dimensions or multiple representations have to be considered. In the segmentation process, there is also the favour of combination of multiple methods in getting the appropriate output. With ensemble of multiple methods, there is a tendency of multiple optimization and decision making process where multiple validity assessment should be used.

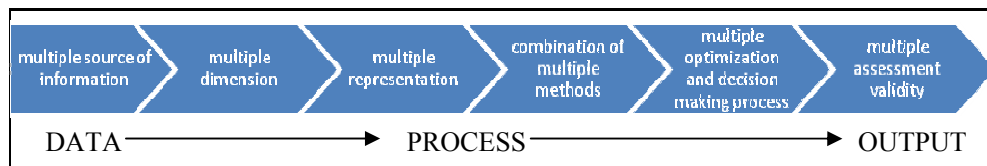


Fig. 2 Consideration of multiple objectives (criteria) from input to output of image segmentation problem

The most commonly used multiobjective optimization method is to transform the multiple objectives into a single-objective function. This is typically done by assigning a numerical weight to each objective (evaluation criterion) and then combining the values of the weighted criteria into a single value by either adding or multiplying all the weighted criteria. Despite transforming multiple objectives into a single-objective optimization problem, another popular method is

simultaneous optimization of several objectives, or called Pareto approaches. In the Pareto approach, the evaluation methods is not simply to aggregate the multiple objectives into an objective functions with a weighted formula. The evaluation of possible (intermediate) solutions is based on dominancy relation [12-14]. This method considers the interrelationship between objectives in evaluating intermediate solution before an algorithm settles to approximately Pareto-optimal solutions [15, 16]. A Pareto-optimal set is a set of solutions that are non-dominated with respect to each other. While moving from one Pareto-optimal solution to another, there is always a certain amount of sacrifice in one objective(s) to achieve a certain amount of gain in the other(s). Pareto-optimal solution sets are often preferred to single solutions because they can be practical when considering real-life problems since the final solution of the decision-maker is always a trade-off [17].

With the consideration of image segmentation as multiple objectives problem, the following section will discuss the current applications of this approach to image segmentation.

3 Overview of current applications that use approaches based on multiple objectives (MO)

The earliest use of MO approach in image segmentation is found in [18]. Image used for analysis was outdoor TV imagery. After this attempt, the use of MO approaches has been found active in image segmentation methods with clustering, classification and histogram thresholding methods. There is also an attempt of using multiobjective approach for evaluation of image segmentation methods. Table 1 summarizes the various types of image dataset used in the current applications of MO approach.

Table 1: The various types of datasets used in current applications

Simulated images	Medical-related images (CT, MRI etc)	Remote sensing images	Natural images
[3] [6] [19] [20] [21] [22] [23] [24] [25] [94]	[3] [6] [22] [26] [27] [28] [29] [30] [31-33] [34] [35]	[36-38] [22] [39] [4, 40, 41] [39] [29] [30] [42] [43-45] [46, 47]	[18, 48] [6] [22] [7] [49] [29] [50] [32] [51] [52] [25, 53] [5, 54] [55] [35] [46, 47]

Simulated Data – include word images and handcrafted image

As shown in Table 1, MO approaches are popular in problems related to medical-related images and remote sensing images. As the nature of these two types of imaging problem is related to multi-spectra image processing, problem

formulation with MO is particularly suitable for combining images with various spatial, spectral and temporal resolutions to form new images [56]. Besides, there is quite a number of related works on feature fusion for the optimization of multiple feature spaces into optimal space, with optimal weight parameters of each feature space. Two examples include the fusion of multiple images' features into one feature representation [25, 57] and fusion of wavelet decomposed subbands into fused transform image [56]. For experimental purposes, there are quite a number of online dataset available for use of experiments in MO methods. The most popular repository is the UCI repository [58] with images related to Dermatology, Digits, Iris, Wine, Wisconsin (Breast cancer), Zoo, Yeast and so on. Other available medical repositories include the BrainWeb [59] and so on. Quite a number of image segmentation approaches have been tested with simulated data or handcrafted data [3, 19-22, 60-63]. Some researchers used multiple datasets to compare their design of MO method. For instance, in literature [3], all types of datasets have been tested to show how generic its MO method in solving image segmentation problem.

Image segmentation can be treated as a pixel classification problem. This classification was conducted by measuring a set of features at each point and defining a decision surface in the feature space [64]. In the classification method, partitioning of object space is exemplarily defined for training objects by the supervisor. The typical question asked is "How to generalize the partition to new objects?" In a classification problem, MO approach has been used in designing the classifier for feature extraction. For instance, in medical image segmentation, classifiers are "trained" to "learn" the boundaries of various tissues in the image. Input data will typically be some subset of images with various intensities value at each spatial location in the image and a vector of features can be constructed from them [65]. The MO approaches have also been used in the design of multiple classifiers and their ensemble in image classification problem [23, 30, 31]. In these classification approaches, some of them are fully supervised and semi-supervised methods. Section 4 in this article will discuss more examples in detail.

Clustering is an unsupervised classification method to group a given collection of unlabeled patterns into meaningful clusters. The applicability of clustering methodology to the image segmentation problem was recognized over three decades ago, and the paradigms underlying the initial pioneering efforts are still in use today. Clustering method is accomplished by unsupervised partitioning of object space by a predefined list of quality criteria such as spatial coherence and feature homogeneity. Thus, the question usually asked is "How to optimize these criteria?" The clustering combination methods are discussed in section 5.

Thresholding is a common region segmentation method [66]. In this technique a threshold is selected, and an image is divided into groups of pixels having values less than the threshold and groups of pixels with values greater or equal to the

threshold. As compared to multiobjective clustering and classification approaches, there is limited research endeavour of using methods with MO in classical histogram thresholding method. In 2007, Nakid and his team [54] have proposed to use the multiobjective approach to find the optimal thresholds of three criteria: the within-class criterion, the entropy and the overall probability of error criterion. The optimization process has been conducted by a new variant of simulated annealing to solve the Gaussian curve-fitting problem. There are also other less popular applications of approaches with MO in region-based segmentation like thresholding method, shape-based segmentation or even to the evaluation model of image segmentation. Although those research activities are rare but it is possible research area for the application of multiobjective optimization. The details of the creative use of MO approaches in these methods are discussed in section 6.

4 Classification techniques for Image Segmentation – MO perspectives

There are two main approaches to image classification: supervised and unsupervised. In the unsupervised approach, the classes are unknown and the approach starts by partitioning the image data into groups (or clusters). According to a similarity measure, the result may be compared with reference to data by an analyst. Therefore, unsupervised classification is also referred to as a clustering problem [67] and it has been discussed in the previous section. In the supervised approach, the number and the numerical characteristics (e.g. mean and variance) of the classes in the image are known in advance and used in the training step which is followed by the classification step [65].

In the use of MO optimization in classification methods, the lists of objective criteria defined are different than the clustering mechanism. In clustering mechanism, usually cluster validity measure indices are formulated as objective functions, but in this perspective, objective functions usually related to the rules definition of the classifiers, error rate of the classifier or diversity measurement. For instance, in [50], the objective functions defined are the tradeoff between the accuracy and diversity of the classifier defined. Similarly, Cococcioni et al. [31] have applied an evolutionary three-objective optimization algorithm to generate an approximation of Pareto-optimal solution set with trade-offs between accuracy and complexity of the classifiers. The applications of MO optimization methods are the most popular in the design of multiple classifiers [23, 30-32, 50-52]. Other applications are found in semi-supervised classification [43-45] and supervised methods [29, 42].

There are usually two layers/levels in the design of the multiple classifiers. For example, in [30], the first level generates a set of good classifiers based on the aggregated error in each separate class and the second level searches the best

ensemble among these classifiers with a multiobjective genetic algorithm. In [29], the first level uses multi-layer perceptron (MLP) neural networks as classifiers to generate a set of classifiers and the second level uses hidden Markov models (HMM) to choose the best team of classifiers.

The ensemble of classifiers has been used to reduce uncertainty of classification model and improve generalization performance [23]. It has been demonstrated that a good ensemble is one where the individual classifiers in the ensemble are both accurate and make their errors on different parts of the input space [23, 50]. In other words, an ideal ensemble consists of good classifiers (not necessarily excellent) that disagree as much as possible on difficult cases. Diversity and accuracy are two important objective criteria are two key issues that should be taken care of, when constructing ensembles [35, 68]. For example, after creating classifiers based on the amount of error created for each class, Ahmadian et al. [30] have taken size, accuracy and two other diversity measures in their use of NSGA II-based algorithm for choosing the best ensembles. Ishibuchi and Nojima [32] have also examined the performance of three multiobjective ensemble classifiers and concentrated on generating an ensemble of classifiers with high diversity. On the other hand, to avoid choosing from overfitting solution, Oliveira et al. [23] have used diversity jointly with the accuracy of the ensemble as selection criterion. Previous research has shown that an ensemble is often more accurate than any of the single classifiers in the ensemble [69, 70]. Although there are several studies in the accuracy and diversity, multi-objectivity in ensembles is still an important area of research that should be explored extensively.

The processing of images that possess ambiguities is better performed using fuzzy segmentation techniques, which are more adept at dealing with imprecise data [71]. Fuzzy rule based image segmentation techniques are able to integrate expert knowledge and are less computationally expensive, compared to fuzzy clustering [71-73]. They are also able to interpret linguistic as well as numeric variables. Therefore, the use of fuzzy rule-based classification system with MO methods is another important research area. For example, a number of non-dominated rule sets have been found from candidate fuzzy rules using an evolutionary multiobjective algorithm [74]. The number of fuzzy rules has been used as a complexity measure, while the number of correctly classified training patterns has been used as an accuracy measure. Ishibuchi and Nojima [32, 51] have tested a similar evolutionary multiobjective fuzzy rule-based classifier in image dataset. Similarly, Cococcioni et al. [31] have applied an evolutionary three-objective optimization algorithm composed of fuzzy rule-based binary classifiers. In Pulkkinen and Koivisto [29], a fuzzy classifier (FC) was obtained by transformation of a decision tree (DT) into FC. Then, the rest of the population was created by randomly replacing some parameters of that FC, in a way that the population was widely spread. That improved the convergence of multiobjective evolution algorithms.

Meanwhile, multiobjective fuzzy clustering scheme has been combined with artificial neural networks (ANN) based probabilistic classifier to yield better performance in [37]. The idea of designing neural networks within a multiobjective setup was first considered by Kottathra and Attikiouzel [75] where they used a branch and bound method to determine the number of hidden neurons (the second objective being the mean square error) in feed forward neural networks. Recently, Abbass [76] proposed an evolutionary multiobjective neural network learning approach where the multiobjective problem formulation essentially involved setting up of two objectives viz. complexity of the network and the training error.

Besides, Ghoggali et al. [43, 45] applied a multiobjective genetic algorithm for Support Vector Machine classifiers to problems with limited training samples. Ghoggali et al. [44] have extended their research to the use of temporal information provided by the user. In [42], multiobjective genetic algorithm, CEMOGA-classifier is used for designing a classifier that can distinguish the pixels belonging to a class, given its intensity values in multiple bands. The ability of GAs and other evolutionary algorithms for handling complex problems, involving features such as discontinuities, multimodality, disjoint feasible spaces and noisy function evaluations, as well as their population-based nature, makes them possibly well suited for optimizing multiple objectives [46, 47].

While many effective algorithms have been developed for constructing classifiers, no single algorithm has been shown to be either empirically or theoretically better than other algorithms in all scenarios [69]. Utility of other recent MOO techniques like particle swarm optimization, artificial immune systems, scatter search and so on needs to be explored [46, 77].

5 Clustering techniques for Image Segmentation – MO perspectives

Clustering in image segmentation is defined as the process of identifying groups of similar image primitives [63]. These image primitives can be pixels, regions, line elements and so on, depending on the problem encountered. An inherent complication in cluster analysis is the lack of a precise definition for what a cluster is [67, 78]. This results in a large number of clustering algorithms, each one looking for clustering problem based on its own definition or criterion [79]. Moreover, clustering algorithms can find structures (partitions) at various refinement levels with a different number of clusters or cluster densities, depending on their parameter settings [80, 81]. As shown in Figure 3, the same dataset can have more than one clustering results, each one represents its interpretation of the data due to a list of possible reasons such as resolution

adjustments, data manipulations, problem formulations, clustering algorithms or adjustment of parameters in the algorithm.

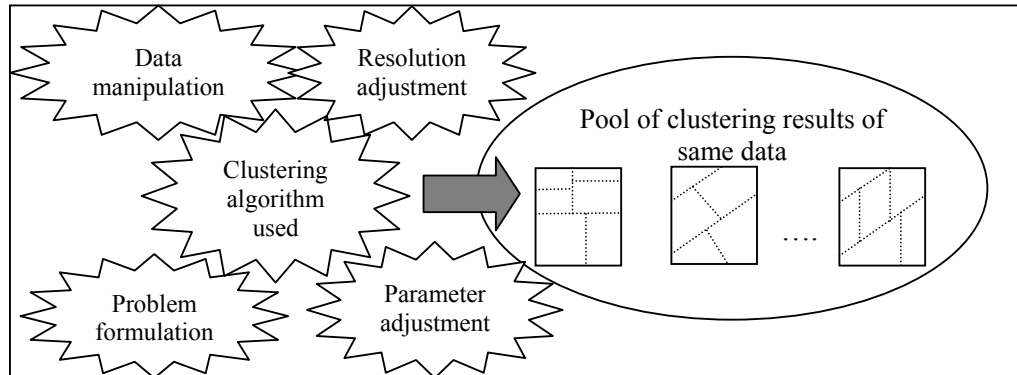


Fig. 3. Possible reasons for a pool of clustering results

In order to compare the current applications that use these MO methods, the authors have summarised in Table 2. The number of objectives considered and the optimization methods used have been identified.

5.1 Clustering approaches related to evolutionary computation algorithms and its variants

In multiobjective image segmentation, the optimization/search techniques used are divided into deterministic and stochastic search techniques [2]. Deterministic search techniques guarantee an optimal partition by performing exhaustive enumeration. Meanwhile, the stochastic search techniques generate a near-optimal partition reasonably quickly, and guarantee convergence to optimal partition asymptotically. Most of the search techniques used in multiobjective image segmentation is evolutionary computation approaches, which is stochastic. Evolutionary computation approaches, motivated by natural evolution, make use of evolutionary operators and a population of solutions to obtain the globally optimal partition of the data [9]. Table 3 lists the basic terminology for evolutionary algorithm. Candidate solutions to the clustering problem are encoded as chromosomes, which usually are made up of real numbers which represent the coordinates of the centers of the partitions in clustering problem [3, 36, 38, 40]. The commonly used evolutionary operators are: selection, recombination, and mutation [67]. Each transforms one or more input chromosomes into one or more output chromosomes. A fitness function evaluated on a chromosome which usually is the objective function that relate to validity measurement, determines a chromosome's likelihood of surviving into the next generation [11].

Table 2: Objective formulation, optimization methods and post-processing in current multiobjective clustering approaches

<i>Ref.</i>	<i>Number of objectives formulated</i>	<i>Optimization method</i>	<i>Post-processing</i>
[26]	Three objective functions	NSGA-II	Three different cluster ensemble techniques [82] are used.
[3]		Simulated annealing	The solution with the minimum Minkowski Score value
[21]	Two objective functions	PESA-II	Model Selection Testing Procedure: The author used the Silhouette Width, Gap statistic or the attainment score method to select the estimated “best” solution and store the Adjusted Rand Index value of the estimated “best” solution.
[20]		SPEA2	Gap statistic
[19]		Evolutionary method	Use meta-clustering algorithm (MCLA) to combine all Pareto-optimal solutions to get the final segmentation.
[40, 41]		Simulated annealing	A measure of indicating the goodness/validity of a cluster solution is used for selection
[4]		Differential Evolution	The solution with the minimum Minkowski Score value
[39]		Particle swarm optimization	The closest solution to the origin of the performance space is selected.
[36]		NSGA-II	A fuzzy voting technique with support vector machine (SVM) classifier
[38] [27]		NSGA-II	None
[7] [49]		PESA-II	
[22]		Differential Evolution	
[28]	Simulated annealing		
[18, 48]	Five objective functions	GA and Hill Climbing	

Table 3: Basic terminology for Evolutionary algorithm and Genetic Algorithm

Term	Description
Individuals	An <i>individual</i> is any point to apply the fitness function. The value of the fitness function for an individual is its score. An individual is sometimes referred to as a <i>genome</i> and the vector entries of an individual as <i>genes</i> .
Representation	Definition of individual.
Populations	A <i>population</i> is an array of individuals. For example, if the size of the population is 100 and the number of variables in the fitness function is 3, the population is represented by a 100-by-3 matrix. The same individual can appear more than once in the population.
Generations	At each iteration, the genetic algorithm performs a series of computations on the current population to produce a new population. Each successive population is called a new <i>generation</i> .
Diversity	The average distance between individuals in a population. A population has high diversity if the average distance is large; otherwise it has low diversity.
Fitness Functions	The function to optimize. For standard optimization algorithms, this is known as the objective function.
Parents and Children	<p>To create the next generation, the genetic algorithm selects certain individuals in the population, called <i>parents</i>, and uses them to individuals in the next generation, called <i>children</i>. Typically, the algorithm is more to select parents that have better fitness</p> <p>The genetic algorithm creates three types of children for the next generation:</p> <ol style="list-style-type: none"> 1. <i>Elite children</i> are the individuals in the current generation with the best fitness values. These individuals automatically survive to the next generation. 2. <i>Crossover children</i> are created by combining the vectors of a pair of parents. 3. <i>Mutation children</i> are created by introducing random changes, or mutations, to a single parent. <p>The schematic diagram illustrates the three types of children.</p> <p>The diagram shows three types of children being created from parents. <ul style="list-style-type: none"> Elite child: A solid black square on the left has an arrow pointing to another solid black square on the right, labeled 'Elite child'. To the right of this is the text 'current create'. Crossover child: Two squares with diagonal lines on the left have arrows pointing to a square with diagonal lines on the right, labeled 'Crossover child'. To the right of this is the text 'likely values'. Mutation child: A circle on the left has an arrow pointing to an oval on the right, labeled 'Mutation child'. </p>
Reproduction	Reproduction controls how the genetic algorithm creates the next generation.
Selection	The selection function chooses parents for the next generation based on their scaled values from the fitness scaling function. An individual can be selected more than once as a parent, in which case it contributes its genes to more than one child.
Elitist selection	The most fit members of each generation are guaranteed to be selected.
Tournament selection	Subgroups of individuals are chosen from the larger population, and members of each subgroup compete against each other. Only one individual from each subgroup is chosen to reproduce.

The popular variants of the evolutionary computation used in multiobjective clustering include NSGA-II [26, 27, 36, 38], SPEA-II [20] and PESA-II [7, 21, 49] (refer Table 2). Non-dominated Sorting Genetic Algorithm (NSGA) suggested by Srinivas and Deb [83] was one of the first evolutionary algorithm. It is based on the creation of an initial random parent population [9]. Individuals selected through a crowded tournament selection undergo crossover and mutation operations to form an offspring population [11]. Both offspring and parent populations are then combined and sorted. Usually, a fixed number of generations will be the termination of the loop. The details of the step-by-step algorithmic design for each of these heuristic methods with genetic algorithm and its variants may be found in [17].

On the other hand, PESA-II [84] is a version of PESA (Pareto Envelope based Selection Algorithm) [85]. In Table 2, [7], [21] and [49] have used PESA-II as the search strategy. Strength Pareto Evolutionary Algorithm 2 (SPEA 2) is a algorithm by Zitzler and Thiele [86]. A density measure is used to discriminate between solutions with the same rank, where the density of a solution is defined as the inverse of the distance to its closest neighbour in objective function space [17]. Another variant of evolutionary method is Differential Evolution (DE) [4, 22] (refer Table 2). DE is a population-based search strategy [87] and its main difference is in the reproduction step where offspring is created from three parents using an arithmetic crossover operator.

5.2 Other MO soft-computing approaches

For image segmentation problem solving, there also exists other soft computing for MO optimization solution as shown in Table 2. The most important among them are the particle swarm optimization (PSO) [39] and simulated annealing approach (SA).

Particle swarm optimization (PSO) is a relatively recent heuristic inspired by the choreography of a bird flock [39, 88]. It improves the exploratory capabilities by introducing a mutation operator whose range of action varies over time. It is unnecessary to perform a fine tuning on the inertia weights used by the expression adopted to compute the velocity of each particle [88]. In [39], a new methodology for clustering hyperspectral images are dealt with a multiobjective PSO.

The simulated annealing approach (SA) used in [3, 28, 40, 41] is a sequential stochastic search technique. Simulated annealing procedures are designed to avoid solutions which correspond to local optima of the objective functions. This is accomplished by accepting a new solution from the next iteration with a lower quality as measured by the criterion function. The acceptance probability is governed by a parameter called the temperature as similar to the annealing process

in metals. It is typically specified in terms of a starting of the first iteration and final temperature value.

On the other hand, Bhanu and his team [18, 48] have designed a hybrid method to combine hill climbing method with genetic algorithm. The hybrid scheme provides performance improvements over the genetic algorithm by taking advantage of both the genetic algorithm's global search ability and the hill climbing's local convergence ability. Hill-climbing is what is known as a greedy algorithm, meaning it always makes the best choice available at each step in the hope that the overall best result can be achieved this way. Besides, in [18, 48], the system employs a scalar evaluation measure which is a weighted combination of the multiple objectives/criteria. Although GA is much similar to evolutionary algorithm but they are different in several ways as shown in Table 4.

Table 4: Differences between Evolutionary algorithm and Genetic Algorithm

Evolutionary algorithm	Genetic Algorithm
It operates on fixed length strings, which contain real values	It operate with binary numbers
Mutation is the driving force.	recombination operator is the primary operator
Selection is deterministic	Selection is probabilistic
It allow self-adaptation, where parameters controlling mutation are allowed to evolve along with object variables	Most popular because they provide a simple framework for attempting to solve complex search problems

6 Miscellaneous applications

Image thresholding is definitely one of the most popular segmentation approaches to extract objects from images for the reason that it is straightforward to implement. It is based on the assumption that the objects can be distinguished by their gray levels. The optimal thresholds are those permitting the distinction of different objects from each other or different objects from the background [48, 89]. However, the automatic fitting of this threshold is one of the main challenges of image segmentation. To overcome this challenge, some researchers have attempted in combining multiple thresholding methods as described in the following subsections. Besides thresholding method, another two interesting applications discussed here include multiobjective shape-based segmentation and segmentation evaluation with MO model.

6.1 Combination of thresholding techniques

Thresholding techniques are image segmentations based on image-space regions. The fundamental principle of thresholding techniques is based on the characteristics of the image [90]. The use of MO optimization in image segmentation with thresholding techniques has been found dominated by Nakid et al. [5, 6, 34, 54]. From 2007 to 2009, they have proposed to find the optimal thresholds that allow to optimize a set of criteria as the objective functions [34, 54]. Their aim is to increase the information on the position of the optimal threshold to obtain the correct segmentation. Combination of segmentation objectives of two classical competing methods: Otsu method and Gaussian curve fitting method has been used. The objective functions for the two methods are optimized and they used weighted sum of within-class criterion and overall probability of error criterion as their objective function. In the first phase, a new peak-finding algorithm is used to identify the most significant peaks in the histogram. In the second phase, they fit the histogram of the image to a sum of Gaussian curves by considering both local and global information. The third phase consists in applying the thresholding process by optimizing the MO function. The issue concerned includes the computational complexity, sensitivity analysis and robustness against noise, visualization viewpoints, object size and image contrast [34].

6.2 Multiobjective shape-based segmentation

The use of MO approach in shape-based segmentation has been conducted by Simari and Singh [91]. They have introduced the notion of multiobjective shape segmentation and the use of multiplicatively weighted Voronoi space partitioning as an approach to segmentation parameterization. The authors proposed seeding approaches to initialize the Voronoi centers, including a novel general-purpose evolutionary approach. They also presented strategies for automatically matching segments to their corresponding labels, including an efficient solution optimal for unary objectives. The method accommodated symmetry constraints which effectively reduce the dimensionality of the optimization domain when prior knowledge of the shape is available. They have used generalized pattern search for their optimization and the user need only provide a constraint function which takes in the non-redundant parameters and produces the others from known symmetry.

6.3 Algorithms Evaluation with Multiobjective model

Another special use of MO approach is for evaluation and comparison of image segmentation algorithms in multi-dimensional fitness spaces by Everingham et. al (2002) [55]. Their area of research is analogous to the use of receiver operating characteristic curves in binary classification problems. Several fitness measures for image segmentation have been proposed. A genetic algorithm dealing MO has been used to explore the set of algorithms, parameters, and corresponding points in fitness space. The principle advantage of this approach is that it avoids the need to aggregate metrics capturing multiple objectives into a single metric, and thus allows trade-offs between multiple aspects of algorithm behavior to be assessed. This is in contrast to previous approaches which have tended to use a single measure of “goodness”, or discrepancy to ground truth data. A modified version of the “Pareto Envelope-based Selection Algorithm” (PESA) has been used to approximate the Pareto-optimal set. PESA is a variant of a genetic algorithm [85], which maintains two populations of chromosomes, with each chromosome encoding an algorithm parameterization. Several fitness functions have been defined and used as cost function for the evaluation functions: (1) Pixel-wise Potential Accuracy, (2) Object-wise Potential Accuracy, (3) Region-wise Information Content and (4) Two other simple cost (negative fitness) functions that measures the mean number of regions per image output by a segmentation algorithm (related to the degree of over- or under-segmentation) and measures the mean processing time per image of the segmentation algorithm.

7 Summary of Design Issues

Image segmentation involves decision making process. In decision making process, three important phases: intelligence, design and choice, should be considered (first column of Figure 4). In the early stage of image segmentation process (second column of Figure 4), decision related to intelligence should be recognized. In this stage, it involves searching the decision environment for conditions calling for decisions; raw data are obtained, processed and examined for clues that may identify opportunities for problems. Therefore, one should concern issues such as problem understanding, goal definition and identification of the possible conflicts between objectives defined.

Next, the design phase involves inventing, developing and analyzing a set of possible solutions or alternative course of action in terms of optimization algorithm to problem identified in the intelligence phase. At this stage, the objective functions for the optimization process should be defined with the selected attributes and appropriate weights values. The optimization/search strategy should be determined with the number of iterations, termination criterion or threshold value. The sensitivity of the parameter adjustment should be justified.

The evaluation of alternative is mainly part of the choice phase. The choice is what many people think of as making a decision. It involves selecting a particular alternative from those available. At this phase, each alternative is evaluated and analyzed in relation to others in terms of a specified rule. The rule is used to rank the alternatives under consideration. The ranking depends on the decision maker's preferences and it can be acquired before or during the optimization process.

The three stages of decision making do not necessarily follow a linear path from intelligence, to design to choice. At any point in the decision making process, it may be necessary to loop back to an earlier phase. For example, one can develop several alternative plans at the design stage but may not be certain whether a specific plan meets the requirements for the decision problem. This requires additional intelligence work. Alternatively, one can be in the process of implementing a decision, only to discover that it is not working, forcing one to repeat the design or choice stage. Each stage of decision-making process requires different types of information.

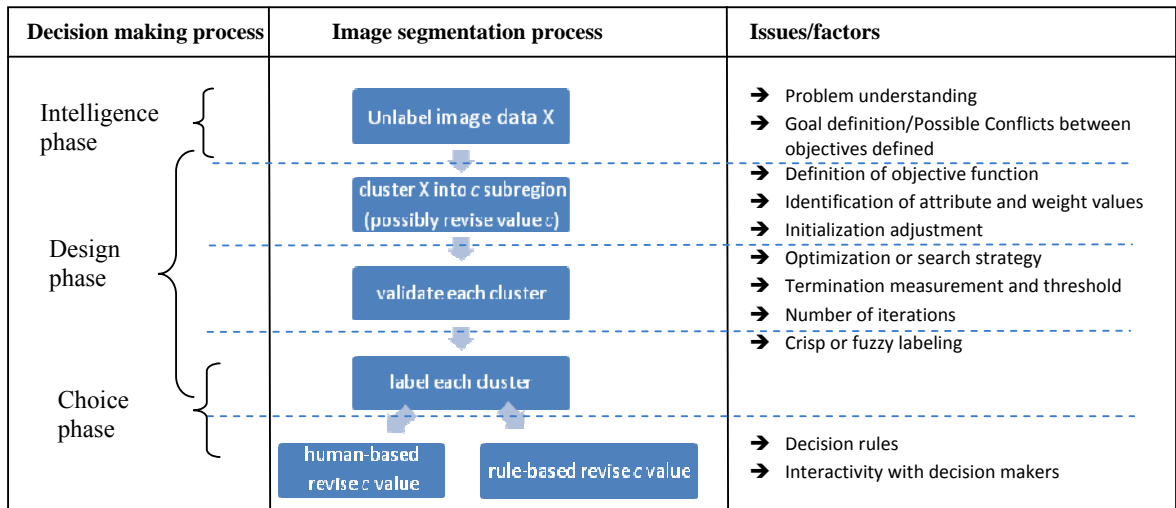


Fig. 4 Image segmentation using approaches coping with MO in three-phases decision making process

A newly designed MO method should provide a realistic solution for developing an image segmentation solution method. In short, the development of a new method should consider factors such as robustness, interactivity, generality, and simplification [13, 16, 92, 93].

8 Conclusion

Most real-world image segmentation problems involve simultaneously optimizing multiple objectives with considerations of the possible trade-offs. For the past ten years, evolutionary approaches and other heuristic methods have been used to solve real-world multiobjective problems. This paper has presented a comprehensive review of the current multiobjective image segmentation approaches by focusing on their components and the salient issues encountered when implementing approaches with MO. Consideration of the computational realities along with the performance of a variety of methods is needed. Also, nearly all problems will require some customization to properly handle the objectives, constraints, encodings and scale. For most implementations, it is not vital to find every Pareto-optimal solution, but rather, efficiently and reliably identify Pareto-optimal solutions across the range of interest for each objective function. In particular, much more work is needed to compare multiobjective image segmentation approaches, both empirically (in a large number of multiple data sets and multiple scenarios) and theoretically.

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APPENDIX

Table A-1. List of some cluster validity measure

Validity index	Description
	*The objective of clustering is to partition the data set X into c clusters
Partition Coefficient (PC)	<p>It measures the amount of "overlapping" between clusters. It is defined as follows:</p> $PC(c) = \frac{1}{N} \sum_{i=1}^c \sum_{j=1}^N (\mu_{ij})^2$ <p>where μ_{ij} is the membership of data point j in cluster i. The disadvantage of PC is lack of direct connection to some property of the data themselves. The optimal number of cluster is at the maximum value.</p>
Classification Entropy (CE):	<p>It measures the fuzziness of the cluster partition only, which is similar to the Partition Coefficient.</p> $CE(c) = -\frac{1}{N} \sum_{i=1}^c \sum_{j=1}^N \mu_{ij} \log(\mu_{ij})$
Partition Index (SC)	<p>It is the ratio of the sum of compactness and separation of the clusters. It is a sum of individual cluster validity measures normalized through division by the fuzzy cardinality of each cluster</p> $SC(c) = \frac{\sum_{i=1}^c \sum_{j=1}^N (\mu_{ij})^m \ x_j - v_i\ ^2}{N_i \sum_{k=1}^c \ v_k - v_i\ ^2}$ <p>SC is useful when comparing different partitions having equal number of clusters. A lower value of SC indicates a better partition.</p>
Separation Index (S)	<p>On the contrary of partition index (SC), the separation index uses a minimum-distance separation for partition validity</p> $S(c) = \frac{\sum_{i=1}^c \sum_{j=1}^N (\mu_{ij})^2 \ x_j - v_i\ ^2}{N \min_{i,k} \ v_k - v_i\ ^2}$
XB index	<p>it aims to quantify the ratio of the total variation within clusters and the separation of clusters</p> $XB(c) = \frac{\sum_{i=1}^c \sum_{j=1}^N (\mu_{ij})^m \ x_j - v_i\ ^2}{N \min_{i,j} \ x_j - v_i\ ^2}$
Silhouette index	<p>Cluster validity index that is used to judge the quality of any clustering solution C. Suppose a represents the average distance of a point from the other points of the cluster to which the point is assigned, and b represents the minimum of the average distances of the point from the points of the other clusters.</p> $s = \frac{b - a}{\max\{a, b\}}$ <p>Silhouette index $s(C)$ is the average Silhouette width of all the data points and it reflects the compactness and separation of clusters. The value of Silhouette index varies from -1 to 1 and higher value indicates better clustering result.</p>
Dunn's index	<p>This index is originally proposed to use at the identification of "compact and well separated clusters". So the result of the clustering has to be recalculated as it was a hard partition algorithm</p> $DI(c) = \min_{i \in c} \left\{ \min_{j \in c, i \neq j} \left\{ \frac{\min_{x \in C_i, y \in C_j} d(x, y)}{\max_{k \in c} \{ \max_{x, y \in C} d(x, y) \}} \right\} \right\}$ <p>The main drawback of Dunn's index is computational since calculating becomes computationally very expansive as c and N increase.</p>