

UTILIZATION OF MULTIVARIATE ADAPTIVE REGRESSION SPLINES (MARS) FOR PREDICTION OF PULL OUT CAPACITY OF SMALL GROUND ANCHOR

Pijush Samui¹, and Ishan Saini²

¹Professor, Centre for Disaster Mitigation and Management, VIT University,
Vellore-632014, India, Tel:91-416-2202283, Fax: 91-416-2243092.
e-mail: pijush.phd@gmail.com

²Undergraduate Student, School of Mechanical and Building Sciences, VIT
University, Vellore-632014, India, Tel: 91-9043293798
email: ishansaini92@gmail.com

Abstract

This article examines the capability of Multivariate Adaptive Regression Spline (MARS) for prediction of pull out capacity (Q) of small ground anchor. MARS is a technique to estimate general functions of high-dimensional arguments given sparse data. The input variables of MARS are anchor diameter (D_{eq}), embedment depth (L), average cone resistance (q_c) along the embedment depth, average sleeve friction (f_s) along the embedment depth and installation technique (IT). Q is the output of MARS. The results of MARS have been compared with the Artificial Neural Network (ANN) model. This study shows that the developed MARS is a robust model for determination of Q of small ground anchor.

Keywords: *Artificial Neural Network; Multi Adaptive Regression Spline; Pull Out Capacity, Small Ground Anchor.*

1 Introduction

Temporary light structures are connected with the ground by small ground anchor. The length of small ground anchor is of about one meter. The pull out capacity (Q) of small ground anchor is of no more than a few kN. The determination of Q of small ground anchor is an imperative task in geotechnical engineering.

Geotechnical engineers use different methods for determination of uplift capacity of ground anchor (Meyerhof and Adams, 1968; Meyerhof, 1973; Meyerhof, 1973; Rowe and Davis, 1982; Rowe and Davis, 1982; Murray and Geddes, 1987; Subba Rao and Kumar, 1994; Basudhar and Singh, 1994; Koutsabeloulis and Griffiths, 1989; Vesic, 1971; Das and Seeley, 1975; Das, 1978; Das, 1980; Das, 1987; Vermeer and Sutjiadi, 1985; Dickin, 1988; Sutherland, 1988). Shahin and Jaksa (2006) have successfully applied Artificial Neural Network (ANN) for determination of Q of small ground anchor. ANN has been successfully used for solving different problems in engineering (Idris et al., 2009; Kuok et al., 2011). However, ANN as a method has some inherent limitations such as black box approach, slow convergence speed, arriving at local minima, low generalization capability, overfitting problem, etc (Park and Rilett, 1999; Kecman, 2001). In view of these deficiencies, this study looked into an alternative approach to estimate Q using the same database. Specifically, this study examined the potential of Multivariate Adaptive Regression Spline (MARS) to predict Q of small ground anchor. MARS is a flexible, more accurate, and faster simulation method for both regression and classification problems (Friedman, 1991; Salford Systems, 2001). It is capable of fitting complex, nonlinear relationships between output and input variables. Researchers have successfully applied MARS for solving different problems in civil engineering (Lall et al., 1996; Attoh-Okine et al., 2003; Attoh-Okine et al., 2009). This study uses the database collected by Shahin and Jaksa (2006). The dataset contains information about equivalent anchor diameter (D_{eq}), embedment depth (L), average cone resistance (q_c) along the embedment depth, average sleeve friction (f_s) along the embedment depth and installation technique (IT). The paper has following aims:

- To examine the feasibility of MARS model for prediction of Q of small ground anchor
- To determine an equation for prediction of Q based on the developed MARS
- To make a comparative study between MARS and ANN model developed by Shahin and Jaksa (2006).

2 Details of Mars

The MARS model splits the data into several splines on an equivalent interval basis (Friedman, 1991). In every spline, MARS splits the data further into many subgroups. Several knots are created by MARS. These knots can be located between different input variables or different intervals in the same input variable, to separate the subgroups. The data of each subgroup are represented by a basis function (BF). The general form of a MARS predictor is as follows:

$$f(x) = \beta_0 + \sum_{j=1}^P \sum_{b=1}^B [\beta_{jb}(+) \text{Max}(0, x_j - H_{bj}) + \beta_{jb}(-) \text{Max}(0, H_{bj} - x_j)] \quad (1)$$

Where x =input, $f(x)$ =output, P = predictor variables and B =basis function. $\text{Max}(0, x-H)$ and $\text{Max}(0, H-x)$ are BF and do not have to each be present if their β coefficients are 0. The H values are called knots. The MARS algorithm consists of (i) a forward stepwise algorithm to select certain spline basis functions, (ii) a backward stepwise algorithm to delete BFs until the “best” set is found, and (iii) a smoothing method which gives the final MARS approximation a certain degree of continuity. BFs are deleted in the order of least contributions using the generalized cross-validation (GCV) criterion (Craven and Wahba, 1979). The GCV criterion is defined in the following way:

$$GCV = \frac{\frac{1}{N} \sum_{i=1}^N [y_i - f(x_i)]^2}{\left[1 - \frac{C(B)}{N}\right]^2} \quad (2)$$

Where N is the number of data and $C(B)$ is a complexity penalty that increases with the number of BF in the model and which is defined as:

$$C(B) = (B + 1) + dB \quad (3)$$

Where d is a penalty for each BF included into the model. It can be also regarded as a smoothing parameter. Friedman (1991) provided more details about the selection of the d . This article adopts the above MARS model for prediction of Q of small ground anchor. The input variables of MARS are D_{eq} , L , q_c , f_s and IT . Q is the output of MARS. Table 1 shows the different statistical parameters of the dataset. The data is normalized between 0 and 1. In order to develop MARS, the data are divided into two groups:

(a) *A training dataset*: This is required to construct the MARS model. In this study, 83 out of the possible 119 cases of small ground anchor are considered for training dataset.

(b) *A testing dataset*: This is required to estimate the MARS model performance. In this study, the remaining 36 data are considered as testing dataset. The MARS model has been developed by using MATLAB.

Table 1. Statistical parameter of the dataset.

<i>Variables</i>	<i>Mean</i>	<i>Standard Deviation</i>	<i>Skewness</i>	<i>Kurtosis</i>
$D_{eq}(\text{mm})$	30.80	7.70	0.93	2.29
$L(\text{mm})$	578.15	118.72	0.03	2.77
$q_c(\text{MPa})$	1.93	00.57	0.51	2.85
$f_s(\text{kPa})$	57.58	40.45	1.83	6.27
IT	1.58	0.49	-0.35	1.12
$Q(\text{kN})$	1.75	0.77	0.18	2.61

3 Results and Discussion

Coefficient of Correlation(R) has been adopted to assess the performance of the developed MARS. The value of R has been determined by using the following relation:

$$R = \frac{\sum_{i=1}^n (Q_{ai} - \bar{Q}_a)(Q_{pi} - \bar{Q}_p)}{\sqrt{\sum_{i=1}^n (Q_{ai} - \bar{Q}_a)^2} \sqrt{\sum_{i=1}^n (Q_{pi} - \bar{Q}_p)^2}} \quad (4)$$

Where Q_{ai} and Q_{pi} are the actual and predicted Q values, respectively, \bar{Q}_a and \bar{Q}_p are mean of actual and predicted Q values corresponding to n patterns. For good model, the value of R should be close to one. Figure 1 shows the flow chart of MARS for prediction of Q .

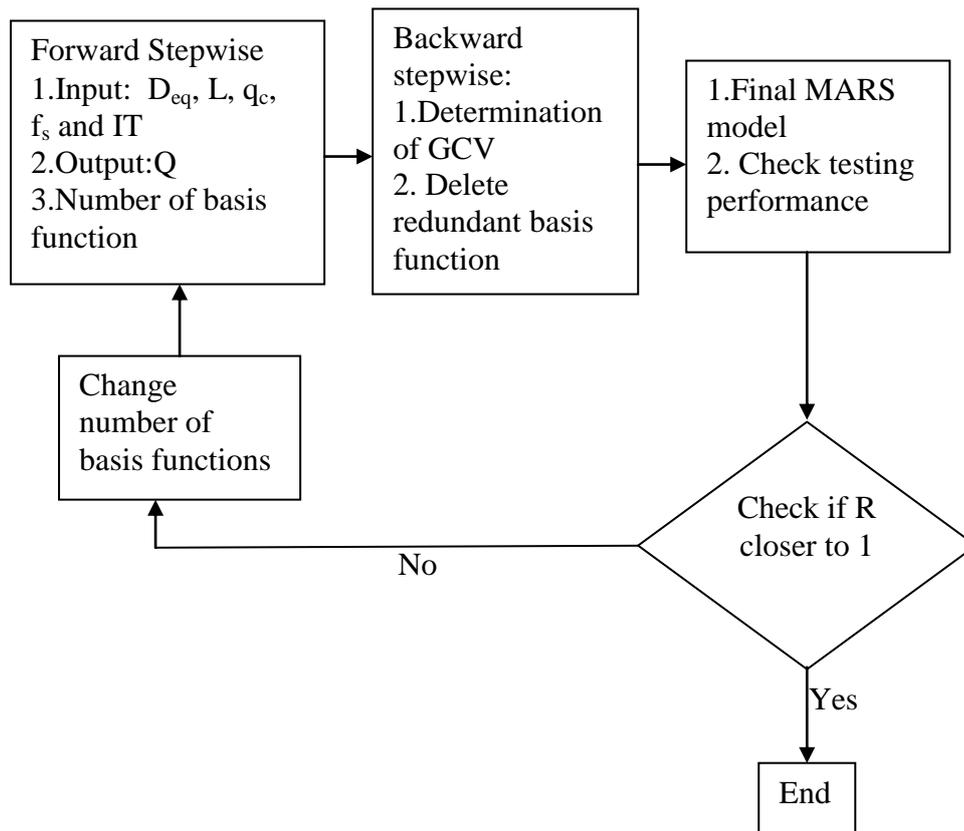


Figure 1. Flow chart of the MARS.

The effect of number basis functions on testing performance has been shown in figure 2.

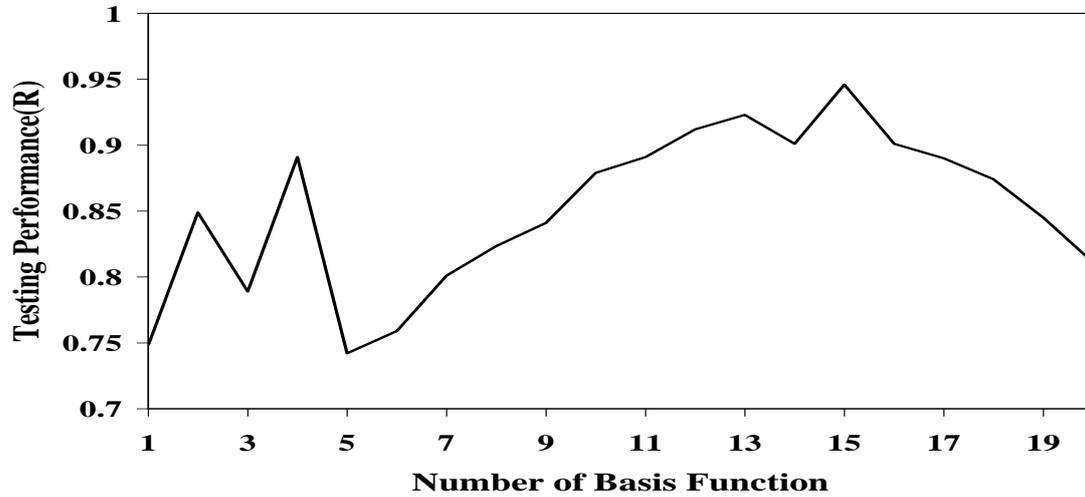


Figure 2. The effect of number of basis function on testing performance in forward stepwise procedure.

It is clear from figure 2 that 15 basis functions give best testing performance in forward stepwise procedure. So, the forward stepwise procedure was carried out to select 15 basis functions (BF) to build the MARS model. This was followed by the backward stepwise procedure to remove redundant basis functions. The final model includes 10 basis functions, which are listed in Table 2 together with their corresponding equations.

Table 2. Basic function and their corresponding equation.

Basis Function	Equation
BF1	$\max(0, f_s - 0.251)$
BF2	$\max(0, 0.251 - f_s)$
BF3	$\max(0, 0.5 - L)$
BF4	$\max(0, L - 0.5) * \max(0, q_c - 0.280)$
BF5	$\max(0, L - 0.5) * \max(0, 280 - q_c)$
BF6	$\max(0, 0.411 - q_c)$
BF7	$BF6 * \max(0, f_s - 0.238)$
BF8	$BF6 * \max(0, 0.238 - f_s)$
BF9	$BF2 * \max(0, L - 0.5)$
BF10	$\max(0, 1 - IT)$

The final equation for the prediction of OCR based on MARS model is given below:

$$Q = 0.533 - 0.169 * BF1 - 1.532 * BF2 - 0.280 * BF3 + 2.228 * BF4 + 1.662 * BF5 - 0.321 * BF6 - 3.145 * BF7 + 1.518 * BF8 - 7.791 * BF9 + 0.065 * BF10 \quad (5)$$

The performance of training and testing dataset has been determined by using the equation (6). Figure 3 illustrates the performance of training dataset.

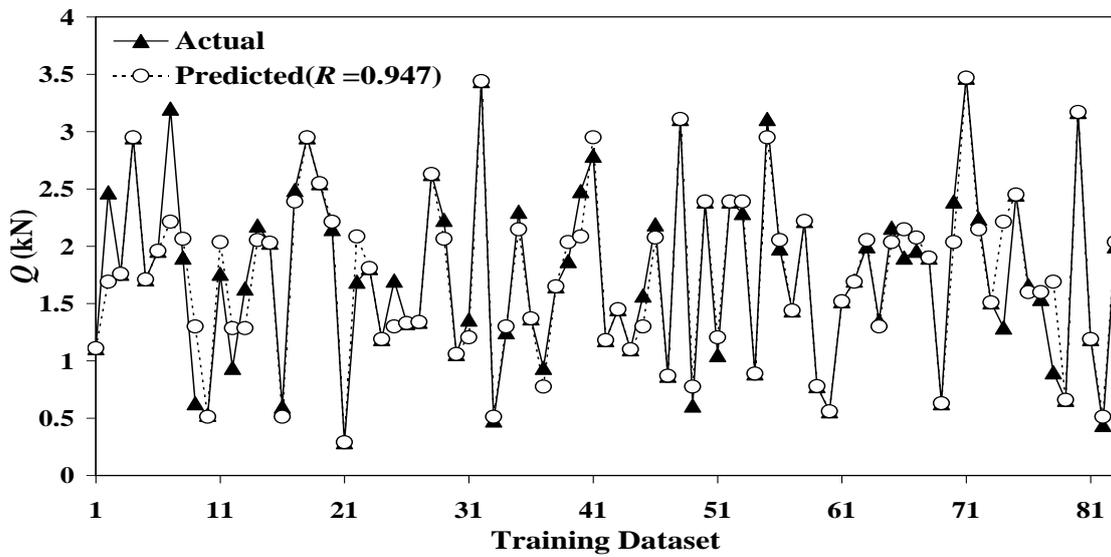


Figure 3. Performance of training dataset.

It is observed from figure 3 that the value of R is close to one. Therefore, the performance of MARS is encouraging for training dataset. The performance of testing dataset has been shown in figure 4.

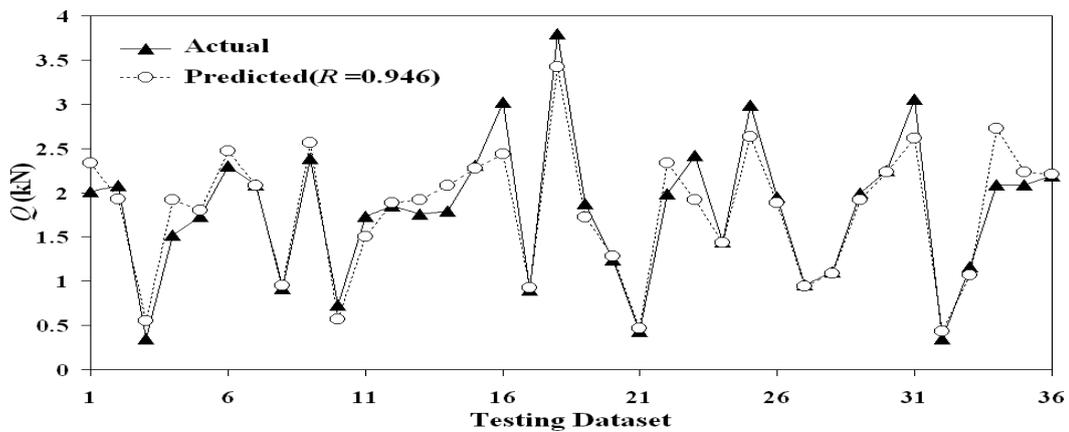


Figure 4. Performance of testing dataset.

Figure 4 also shows that the value of R is close to one. So, the developed MARS has ability to predict Q of small ground anchor. A comparative study has been carried out between developed MARS and ANN model developed by Shahin and Jaksa (2006). Comparison has been done for testing dataset. Table 3 shows the value of Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) for MARS and ANN models.

Table 3. Comparison between ANN and MARS models.

Model	RMSE(kN)	MAE(kN)
ANN	0.3971	0.3097
MARS	0.2490	0.1891

The values of RMSE and MAE have been determined by using the following relation.

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (Q_{ai} - Q_{pi})^2}{n}} \quad (6)$$

$$MAE = \frac{\sum_{i=1}^n |Q_{ai} - Q_{pi}|}{n} \quad (7)$$

It is observed from table 2, the performance of MARS is better than the ANN model. The generally low RMSE and MAE, high R, and simple algorithms demonstrate the promise of MARS models in accurate prediction of Q of small ground anchor. The performance of MARS models can be improved with more data collection.

4 Conclusion

This study used the MARS approach for prediction of Q of small ground anchor. The results show that the developed MARS can accurately predict Q of small ground anchor. The performance of the developed MARS is better than the ANN model. User can use the developed equation for practical purpose. The developed MARS model automatically selects the parameters and the structure of the model based on data available. It is concluded that the MARS technique is an effective tool for prediction of Q of small ground anchor.

References

- [1] A.S. Vesic, "Breakout resistance of objects embedded in ocean bottom", *J. Soil Mech. And Found. Div., ASCE* 96 (1971) 1311–1334.
- [2] B.M. Das, "A procedure for estimation of ultimate uplift capacity of foundations in clay", *Soils and Foundations* 20 (1980) 77–82.
- [3] B.M. Das, "Developments in geotechnical engineering", *Theoretical Foundation Engineering*, Elsevier, 1987.
- [4] B.M. Das, "Model tests for uplift capacity of foundations in clay", *Soils and Foundations* 18 (1978) 17–24.
- [5] B.M. Das, G.R. Seeley, "Breakout resistance of horizontal anchors", *J. Geotech. Engrg. Div. ASCE* 101 (1975) 999–1003.
- [6] D. Park, L.R. Rilett, "Forecasting freeway link travel times with a multi-layer feed forward neural network", *Computer Aided Civil and Infra Structure Engineering* 14 (1999) 358 – 367.
- [7] E.A. Dickin, "Uplift behaviour of horizontal anchor plates in sand", *J. Geotech. Engrg. Div. ASCE* 114 (1988) 1300–1317.
- [8] E.J. Murray, J.D. Geddes, "Uplift of anchor plates in sand", *J. Geotech. Engrg. Div. ASCE*. 113 (1987) 202–215.
- [9] G.G. Meyerhof, "Uplift resistance of inclined anchors and piles", *In: Proc., 8th Int. Conf. on Soil Mechanics and Foundation Engg.*, Moscow, USSR, 1973.
- [10] G.G. Meyerhof, J.I. Adams, "The ultimate uplift capacity of foundations", *J. Canadian Geotech*, 5 (1968) 225–244.
- [11] H.B. Sutherland, "Uplift resistance of soils", *Geotechnique* 38 (1988) 473–516.
- [12] J.H. Friedman, "Multivariate adaptive regression splines", *Ann Stat.* 19 (1991) 1–141.
- [13] K. K. Kuok, S. Harun, C.P. Chan, "Hourly Runoff Forecast at Different Lead-time for a Small Watershed using Artificial Neural Networks", *Int. J. Advance. Soft Comput. Appl.*, 3 (1) (2011) 68-86.
- [14] K.S. Subba Rao, J. Kumar, "Vertical uplift capacity of horizontal anchors", *J. Geotech. Engrg. Div. ASCE*. 120 (1994) 1134–1147.
- [15] M.A. Shahin, M.B. Jaksa, "Pullout capacity of small ground anchors by direct cone penetration test methods and neural networks", *J. Canadian Geotech*. 43 (2006) 626-637.
- [16] N. Idris, N. Yusof, P. Saad, "Adaptive Course Sequencing for Personalization of Learning Path Using Neural Network", *Int. J. Advance. Soft Comput. Appl.*, 1 (1) 2009 .
- [17] N.C. Koutsabeloulis, D.V. Griffiths, "Numerical modeling of the trap door problem", *Geotech*. 39 (1989) 77–89.

- [18] N.O. Attoh-Okine, K. Cooger, S. Mensah, “Multivariate adaptive regression (MARS) and hinged hyperplanes (HHP) for doweled pavement performance modeling”, *Construction and Building Materials* 23 (2009) 3020-3023.
- [19] N.O. Attoh-Okine, S. Mensah, M. Nawaiseh, “A new technique for using multivariate adaptive regression splines (MARS) in pavement roughness prediction”, *Proceedings of the ICE – Transport* 156 (2003) 51–55.
- [20] P. Craven, G. Wahba, “Smoothing noisy data with spline functions: estimating the correct degree of smoothing by the method of generalized cross-validation”, *Numer. Mat.* 31 (1979) 317–403.
- [21] P.A. Vermeer and W. Sutjiadi, “The uplift resistance of shallow embedded anchors”, *In: Proc., 11th Int. Conf. Soil Mech. and Found. Engrg.*, San Francisco, Calif., 1985.
- [22] P.K. Basudhar, D.N. Singh, “A generalized procedure for predicting optimal lower bound break-out factors of strip anchors”, *Geotech*, 44 (1994) 307–318.
- [23] R.K. Rowe, E.H. Davis, “The behaviour of anchor plates in clay”, *Geotech.* 32 (1982) 9–23.
- [24] R.K. Rowe, E.H. Davis, “The behaviour of anchor plates in sand”, *Geotech.* 32 (1982) 25–41.
- [25] Salford Systems, *MARSTM User Guide*, Salford Systems, San Diego, California, USA, 2001.
- [26] U. Lall, T. Sangoyomi, H.D.I. Abarbanel, “Nonlinear dynamics of the Great Salt Lake: Nonparametric short term forecasting”, *Water Resour. Res.* 32 (1996) 975–985.
- [27] V. Kecman, *Learning and Soft Computing: Support Vector Machines, Neural Networks, and Fuzzy Logic Models*, The MIT press, Cambridge, Massachusetts, London, England, 2001.