Int. J. Advance. Soft Comput. Appl., Vol. 6, No.3, November 2014
ISSN 2074-8523

A Mean Potentiality approach of an Intuitionistic Fuzzy Soft Set Based Decision Making Problem in Medical Science

P. Muthukumar[1], G. Sai Sundara Krishnan[2]

[1]Assistant Professor (SG), Department of Mathematics,
PSG College of Technology, Coimbatore, Tamil Nadu - 641 004, India.
e-mail: muthushravan@gmail.com

[2]Associate Professor, Department of Applied Mathematics and Computational Sciences, PSG College of Technology, Coimbatore, Tamil Nadu - 641 004, India.
e-mail: g_ssk@yahoo.com

Abstract

In this paper, a Mean Potentiality Approach for a balanced solution of an Intuitionistic Fuzzy Soft Set (IFSS) based decision making problem is proposed using level soft sets. Further, a parameter reduction procedure has been used to reduce the choice parameter set with the help of the balanced algorithm of mean potentiality approach. Moreover, we implement this mean potentiality approach of a balanced solution of an IFSSs in medical diagnosis has been exhibiting with a hypothetical case study.

Keywords: *Soft set, Fuzzy Soft set, Intuitionistic Fuzzy Soft set, Mean Potentiality Approach.*

1 Introduction

Most of our real life problems in economics, engineering, environment, social science and medical science, etc., involve imprecise data. To handle such situations, consider some of the theories, such as probability theory, fuzzy theory, rough sets theory and interval mathematics. Although these theories can successfully be used to extract useful information hidden in imprecise data, each of them has its inherent difficulties. In 1999, Molodtsov [3] proposed soft sets as a completely generic mathematical tool for modeling uncertainties, which is free from the difficulties affecting the existing methods. Maji et al. [19] defined fuzzy soft sets, combining soft sets with fuzzy sets. Maji et al. [16, 18] reported a detailed theoretical study and decision making problems on soft sets. Moreover, Maji et al. [17, 20] extended the concept of soft sets to intuitionistic fuzzy soft sets from Atanassov intuitionistic fuzzy sets [10]. N.Cagman et al. [9, 12, 13] initiated the notion of soft groups, soft matrix theory and uni-int decision making problems. Majumdar et al. [14] extended fuzzy soft sets into generalized fuzzy soft sheets and [15] proposed a new student ranking system based on generalized fuzzy soft set theory. D. Chen et al [2] and Z. Kong et al. [31] introduced a new definition of parameter and normal parameter reduction into soft sets. Further investigated by several researchers [11, 22, 23, 24, 26, 27, 28, and 30].

Feng et al. [4, 7, 8] investigated the relationship among soft sets, soft rough sets and soft rough fuzzy sets, also presented a novel approach of soft semi rings in [6]. Jiang's et al. [25, 29] introduced an adjustable approach to fuzzy soft set based decision making by means of level soft sets. But according to Jiang's method, the decision maker can select any level to form the level soft set. There does not exist any unique or uniform criterion for the selection of the level. So by this method the decision maker cannot decide which level is suitable to select the optimal choice object. Recently, T. Mitra Basu et al. [21] have proposed mean potentiality approach to enhance the Feng's method [5] to get the better and unique criterion on a balanced solution of a fuzzy soft set based decision making problem.

In this paper, the mean potentiality approach is extended for IFSSs based decision making problems. This concept of IFSSs is more realistic as it contains a degree of both membership and non-membership corresponding to each parameter. This technique has been tested on various data sets from [32] and a comparison has been made with the Jiang's method. It has completely different from the existing methods presented in [5, 29].

The organization of the rest of the paper is as follows: In section 3, we focus optimality criteria for a balanced solution of an IFSSs based decision making problems and introduced the definition of the measure of performance of IFSSs. Also to address about the limitations of Jiang's method by illustrate with an example. We developed the concept of mean potentiality approach and an algorithm based on this approach to get a balanced solution of an IFSSs based decision making problems in section 4. In section 5, a parameter reduction procedure has been used to reduce the choice parameter set with the help of the balanced algorithm of mean potentiality approach. In section 6, we demonstrate the application of the above two algorithms in the medical diagnosis problem to find out the optimal disease and also compared with the Jiang's method. Finally, we present an adjustable approach to weighted IFSSs based decision making problems by extending the approach to weighted fuzzy soft sets based decision making.

2 Related Works

In this section, we present brief preliminaries on the theory of soft sets, fuzzy soft sets, intuitionistic fuzzy soft sets and level soft sets.

Definition 2.1 [3] *Let U be an initial universe set and E be a set of parameters. A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U.*

In other words, the soft set is a parameterized family of subsets of the set U.

Definition 2.2[19] *Let U be an initial universe set and E be a set of parameters.* Let P (U) denotes the set of all fuzzy sets of U. Let A ⊂ E. A pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by F: A→ P (U).

Definition 2.3[20] *Let U be an initial universe set and E be a set of parameters.* Let IF^U denotes the collection of all intuitionistic fuzzy subsets of U. Let A⊂E. A pair (F, A) is called intuitionistic fuzzy soft sets (IFSSs) over U, where F is a mapping given by F: A → IF^U.

Definition 2.4[21] *The Parameters of a decision maker's choice or requirement which forms a subset of the whole parameter set of that problem are known as choice parameters.*

Definition 2.5 [21] *Choice value of an object is the sum of the membership values of that object corresponding to all the choice parameters associated with a decision making problem.*

Definition 2.6 [29] *Let $\overline{\omega} = (F, A)$ be an IFSS over U, where A⊂E and E is a set of parameters.*
For $s, t \in [0,1]$, the (s, t)-level soft set of $\overline{\omega}$ is a crisp soft set $L(\overline{\omega}; s, t) = (F_{(s,t)}, A)$ defined by

$$F_{(s,t)}(\varepsilon) = L(F(\varepsilon); s, t) = \{x \in U \mid \mu_{F(\varepsilon)}(x) \geq s \text{ and } \gamma_{F(\varepsilon)}(x) \leq t\} \text{ for all } \varepsilon \in A.$$

Definition 2.7 [29] *Let $\overline{\omega} = (F, A)$ be an IFSS over U, where A⊂E and E is a set of parameters.*
Let $\lambda: A \to [0,1] \times [0,1]$ be an intuitionistic fuzzy set in A which is called a threshold intuitionistic fuzzy set. The level soft set of $\overline{\omega}$ with respect to λ is a crisp soft set $L(\overline{\omega}; \lambda) = (F_\lambda, A)$ defined by

$$F_\lambda(\varepsilon) = L(F(\varepsilon); \lambda(\varepsilon)) = \{x \in U \mid \mu_{F(\varepsilon)}(x) \geq \mu_\lambda(\varepsilon) \text{ and } \gamma_{F(\varepsilon)}(x) \leq \gamma_\lambda(\varepsilon)\}$$
$$\text{for all } \varepsilon \in A.$$

Definition 2.8 [29] *Let* $\bar{\omega} = (F, A)$ *be an IFSS over U, where A⊂E and E is a set of parameters.*
Based on the IFSS set $\bar{\omega} = (F, A)$, *we can define an intuitionistic fuzzy set* $mid_{\bar{\omega}}: A \to [0,1] \times [0,1]$ *by*

$$\mu_{mid_{\bar{\omega}}}(\varepsilon) = \frac{1}{|U|} \sum_{x \in U} \mu_{F(\varepsilon)}(x) \text{ and } \gamma_{mid_{\bar{\omega}}}(\varepsilon) = \frac{1}{|U|} \sum_{x \in U} \gamma_{F(\varepsilon)}(x) \text{ for all } \varepsilon \in A.$$

The intuitionistic fuzzy set $mid_{\bar{\omega}}$ is called the mid-threshold of the IFSS $\bar{\omega}$ and denoted by $L(\bar{\omega}; mid)$.

Definition 2.9 [29] *Let* $\bar{\omega} = (F, A)$ *be an IFSS over U, where A⊂E and E is a set of parameters.*
Based on the IFSS $\bar{\omega} = (F, A)$, we can define an intuitionistic fuzzy set $topbottom_{\bar{\omega}}: A \to [0,1] \times [0,1]$ by

$$\mu_{topbottom_{\bar{\omega}}}(\varepsilon) = \max_{x \in U} \mu_{F(\varepsilon)}(x) \text{ and } \gamma_{topbottom_{\bar{\omega}}}(\varepsilon) = \min_{x \in U} \gamma_{F(\varepsilon)}(x)$$
$$\text{for all } \varepsilon \in A.$$

The intuitionistic fuzzy set $topbottom_{\bar{\omega}}$ is called the topbottom-threshold of the IFSS $\bar{\omega}$ and denoted by $L(\bar{\omega}; topbottom)$.

Definition 2.10 [29] *Let* $\bar{\omega} = (F, A)$ *be an IFSS over U, where A⊂E and E is a set of parameters.*

Based on the IFSS set $\bar{\omega} = (F, A)$, we can define an intuitionistic fuzzy set $toptop_{\bar{\omega}}: A \to [0,1] \times [0,1]$ by

$$\mu_{toptop_{\bar{\omega}}}(\varepsilon) = \max_{x \in U} \mu_{F(\varepsilon)}(x) \text{ and } \gamma_{toptop_{\bar{\omega}}}(\varepsilon) = \max_{x \in U} \gamma_{F(\varepsilon)}(x)$$
$$\text{for all } \varepsilon \in A.$$

The intuitionistic fuzzy set $toptop_{\bar{\omega}}$ is called the toptop-threshold of the IFSS $\bar{\omega}$ and denoted by $L(\bar{\omega}; toptop)$.

Definition 2.11 [29] *Let* $\bar{\omega} = (F, A)$ *be an IFSS over U, where A⊂E and E is a set of parameters.*

Based on the IFSS set $\bar{\omega} = (F, A)$, we can define an intuitionistic fuzzy set $bottombottom_{\bar{\omega}}: A \to [0,1] \times [0,1]$ by

$$\mu_{bottombottom_{\bar{\omega}}}(\varepsilon) = \min_{x \in U} \mu_{F(\varepsilon)}(x) \text{ and}$$
$$\gamma_{bottombottom_{\bar{\omega}}}(\varepsilon) = \min_{x \in U} \gamma_{F(\varepsilon)}(x) \text{ for all } \varepsilon \in A.$$

The intuitionistic fuzzy set $bottombottom_{\bar{\omega}}$ is called the bottombottom-threshold of the IFSS $\bar{\omega}$ and denoted by $L(\bar{\omega}; bottombottom)$.

Definition 2.12 [29] *Let IF(U) be the set of all intuitionistic fuzzy sets in the universe U.*

Let E be a set of parameters and A ⊂ E. A weighted IFSS is a triple $\xi = (F, A, \omega)$, where (F, A) is an intuitionistic fuzzy soft set over U, and $\omega : A \rightarrow [0,1]$ is a weight function specifying the weight $w_j = \omega(\varepsilon_j)$ for each attribute $\varepsilon_j \in A$.

3 Proposed Work

In this section, we proposed an optimality criteria for a balanced solution of IFSSs based decision making problems and introduced a performance measure for IFSSs. Further, the limitations of Jiang's method were discussed through an illustrated example.

Some researchers have worked to get solution of the IFSS based decision making problems with equally weighted choice parameters. According to their methods the selected object may have a considerable difference between the membership as well as the non-membership values of the choice parameters though they are equally weighted. In real life there are many problems in which selection is expected in such a way that all criteria, i.e., choice parameters associated with the selected object will be more or less of same importance, i.e., there will not be any significant difference between the membership values of the selected object from the choice parameters. In this parlance, this paper focus a balanced solution of an IFSS in which all the choice parameters are satisfied mostly and the satisfaction (membership and non-membership values) for every choice parameters are close to each other as much as possible.

3.1 Optimality criteria

To get a balanced solution of an IFSS based decision making problem, with equally weighted choice parameters the following criteria must be satisfied:

a) At least one object satisfies all the choice parameters mostly. In other words, the choice value of at least one object be maximum.

b) At least in one object with maximum choice value, the satisfaction (i.e., membership and non-membership vales) with every choice parameters are almost same. There is not a huge difference between the membership and non-membership from one choice parameter to another. They should be close to each other.

Example

Let (F, P) be an IFSS over U. The tabular representation of IFSSs (F, P) with choice values is given Table 1. It shows that the choice value for the objects O_1, O_2 are same, but the non-negative difference $(\zeta^i_{jk}, \eta^i_{jk})$ between the membership and non-membership values of each object (O_i) associated with the parameters e_j and e_k is given by $\zeta_{jk}^{i} = \left|\mu_{e_j}(O_i) - \mu_{e_k}(O_i)\right|$ and $\eta_{jk}^{i} = \left|\nu_{e_j}(O_i) - \nu_{e_k}(O_i)\right|$.

So in this case, $\zeta_{12}^{1} = 0.1$, $\zeta_{13}^{1} = 0.5$, $\zeta_{23}^{1} = 0.4$ and $\eta_{12}^{1} = 0.1$, $\eta_{13}^{1} = 0.4$, $\eta_{23}^{1} = 0.3$ for O_1 and $\zeta_{12}^{2} = 0.1$, $\zeta_{13}^{2} = 0.2$, $\zeta_{23}^{2} = 0.1$ and $\eta_{12}^{2} = 0$, $\eta_{13}^{2} = 0.1$, $\eta_{23}^{2} = 0.1$ for O_2. Therefore the sum of these differences for O_1 and O_2 are given by

Table 1: Tabular representation of a IFSS (F, P)

	e_1	e_2	e_3	Choice value
O_1	(0.9, 0)	(0.8, 0.1)	(0.4, 0.4)	(2.1, 0.5)
O_2	(0.6, 0.2)	(0.7, 0.2)	(0.8, 0.1)	(2.1, 0.5)

$$\Lambda^1 = \sum_{i=1}^{3}\sum_{(j=1)(i\neq j)}^{3}(\zeta_{jk}^{1}, \eta_{jk}^{1}) = (0.1 + 0.5 + 0.4\ , 0.1 + 0.4 + 0.3) = (1, 0.8)$$

$$\Lambda^2 = \sum_{i=1}^{3}\sum_{(j=1)(i\neq j)}^{3}(\zeta_{jk}^{2}, \eta_{jk}^{2}) = (0.1 + 0.2 + 0.1\ , 0 + 0.1 + 0.1) = (0.4, 0.2)$$

Now Λ^1 is very much larger than Λ^2, which implies that there are far difference between the membership and non-membership values of O_1 for the choice parameters e_1, e_2, e_3. On the other hand, the small value of Λ^2 indicates that satisfaction (membership and non-membership values) of O_2 associated with these same choice parameters are very closed to each other.

Hence, one of the most important optimality criteria for a balanced solution is to minimize this Λ-value.

3.2 Measure of performance

The measure of performance of a method (M) which satisfies the optimality criteria to IFSS based decision making problem is defined as, the sum of the inverse of the summation of the non-negative difference between the membership values of the optimal object for the choice parameters and the choice value of the optimal object and similarly for non-membership values of the optimal object, i.e., it is mathematically defined as,

$$\Upsilon_M = \left(\begin{array}{c} \frac{1}{\sum_{i=1}^{m}\sum_{(j=1)(i\neq j)}^{m}\left|\mu_{e_i}(Op)-\mu_{e_j}(Op)\right|} + \sum_{i=1}^{m}\mu_{e_i}(Op), \\ \frac{1}{\sum_{i=1}^{m}\sum_{(j=1)(i\neq j)}^{m}\left|\nu_{e_i}(Op)-\nu_{e_j}(Op)\right|} + \sum_{i=1}^{m}\nu_{e_i}(Op) \end{array} \right)$$

Where m is the number of choice parameters and $\mu_{e_i}(O_p)$ is the membership value of the optimal object (O_p) and $\nu_{e_i}(O_p)$ is the non-membership value of the optimal object (O_p) for the choice parameter e_i.

Suppose there are two methods M_1 and M_2, which satisfy the optimality criteria and their measure of the performances are respectively Υ_{M_1} and Υ_{M_2}. Now three cases will arise:

i) if $\Upsilon_{M_1} > \Upsilon_{M_2}$ then M_1 be a better method than M_2,

ii) if $\Upsilon_{M_1} < \Upsilon_{M_2}$ then M_2 be a better method than M_1 and

iii) if $\Upsilon_{M_1} = \Upsilon_{M_2}$ then the performance of the both methods be the same.

3.3 Limitations of Jiang's method [28]

Though the Jiang's method is efficient in selecting the optimal object of a IFSSs based decision making problem, with equally weighted parameters, but it possesses some inherent drawbacks. We can illustrate this with the following example.

Example

Let U be the set of three houses, given by, U = {h_1, h_2, h_3}. Let E be the set of parameters (each parameter is a fuzzy word), and is given by, E = {beautiful (e_1), wooden (e_2), modern (e_3), well furnished (e_4), in the green surroundings (e_5), well ventilated (e_6), well situated (e_7) }.

Let the IFSS (F, E) describes the "attractiveness of the houses" and is given by

(F, E) = {beautiful houses $= \{ \frac{h1}{(0.2,0.7)}, \frac{h2}{(0.9,0.05)}, \frac{h3}{(0.4,0.5)} \}$,

Wooden houses $= \{ \frac{h1}{(0,0.8)}, \frac{h2}{(1,0)}, \frac{h3}{(0.3,0.5)} \}$,

Modern houses $= \{ \frac{h1}{(0.8,0.1)}, \frac{h2}{(0.6,0.2)}, \frac{h3}{(0.7,0.2)} \}$,

Well furnished houses $= \{ \frac{h1}{(0.9,0.05)}, \frac{h2}{(0.1,0.8)}, \frac{h3}{(0.5,0.3)} \}$,

With green surroundings $= \{ \frac{h1}{(0,0.6)}, \frac{h2}{(0.8,0.1)}, \frac{h3}{(0.3,0.6)} \}$,

Well ventilated houses $= \{ \frac{h1}{(0.7,0.2)}, \frac{h2}{(0.9,0)}, \frac{h3}{(0.5,0.3)} \}$,

Well situated houses $= \{ \frac{h1}{(0.2,0.6)}, \frac{h2}{(0.7,0.2)}, \frac{h3}{(0.8,0.2)} \}$ }

The set of choice parameters of Mr. X is, P = {modern (e_3), well ventilated (e_6), well situated (e_7)}.Then the tabular representation of (F, P) is given in the Table 2.

Table 2: The tabular representation of (F, P)

	e_3	e_6	e_7
h_1	(0.8, 0.1)	(0.7, 0.2)	(0.2, 0.6)
h_2	(0.6, 0.2)	(0.9, 0)	(0.7, 0.2)
h_3	(0.7, 0.2)	(0.5, 0.3)	(0.8, 0.2)

Using mid level soft set method, the tabular representation of the corresponding mid level soft set of (F, P) with the choice values of the houses is given in Table 3.

Table 3: The tabular representation of L((F, P), mid)

	e_3	e_6	e_7	Choice value
h_1	1	1	0	2
h_2	0	1	1	2
h_3	1	0	1	2

In Table 3, the choice values of all the houses are equal. In this situation, the decision maker may select any one of the houses (according to Jiang's method) as his optimal choice. Suppose that the decision maker select the first house (h_1). But by observation, from Table 3, it should not be the best compared to other two houses (h_2 and h_3). Since for this house (h_1), the third parameter satisfies the membership and non-membership value (0.2, 0.6), which is less than that of the other two houses h_2 and h_3. Moreover, all the choice parameters in this problem have the same weight. So it is necessary to balance among the membership and non-membership values of the choice parameter of the optimal choice house. So, mid -level soft set approach is not suitable for this problem.

By using topbottom and toptop-level soft set approach to choose the best object, the tabular representation of the corresponding topbottom and toptop-level soft set of (F, P) with the choice values of the houses is given in the Table 4 and Table 5 respectively. In these methods also the choice values of all the houses are equal. So, there is no clear indication to choose the level. Hence, the decision maker cannot decide which level is suitable to select a balanced solution of the problem.

Table 4: The tabular representation of L((F, P), topbottom)

	e_3	e_6	e_7	Choice value
h_1	1	0	0	1
h_2	0	1	0	1
h_3	0	0	1	1

Table 5: The tabular representation of L((F, P), toptop)

	e_3	e_6	e_7	Choice value
h_1	1	0	0	1
h_2	0	1	0	1
h_3	0	0	1	1

4 Mean potentiality approach

To overcome the difficulties of Jiang's method, we introduce a mean potentiality approach to obtain a synchronized solution of an IFSS based decision making problem, with equally weighted choice parameters which comprises of some new notions in this section.

Definition 4.1 *The potentiality of an IFSS* (p_{ifs}) *is defined as the sum of all memberships and non-memberships values of all objects with respect to all parameters,*

i.e., mathematically it is defined as

$$p_{ifs} = \left(\sum_{i=1}^{m}\sum_{j=1}^{m}\mu_{ij}\,,\ \sum_{i=1}^{m}\sum_{j=1}^{m}\nu_{ij}\right).$$

Where μ_{ij} and ν_{ij} is the membership and non-membership values of the i^{th} object with respect to j^{th} parameter respectively, m is the number of objects and n is the number of parameters.

Definition 4.2 *The mean potentiality* (m_p) *of IFSSs is defined as its average weight among the total potentiality*,

i.e., mathematically it is defined as

$$m_p = \frac{p_{ifs}}{m\times n}\,.$$

Definition 4.3 *Balance Algorithm*

Now we developed the balanced Algorithm for finding a balanced solution of an IFSSs based decision making problem with equally weighted choice parameters.

Step 1: Find a normal parameter reduction Q of the choice parameter set P. If it exists construct the tabular representation of (F, Q). Otherwise, construct the tabular representation of the IFSSs (F, P) with the choice value of each object.

Step 2: Compute the potentiality $\left(p_{ifs}\right)$ of the IFSS according to our definition.

Step 3: Then find out the mean potentiality (m_p) of the IFSS up to ϱ significant figures (where ϱ is the maximum number of significant figures among all the membership and non-membership values of the object concerned with the problem).

Step 4: Now form m_p–level soft set of the IFSS and represent this in tabular form, then compute the choice value c_i for each object O_i $\forall i$.

Step 5: If c_k is maximum and unique among $c_1, c_2, \ldots c_m$, where m is the number of objects (rows) then the optimal choice object is O_k and then the process will be stopped. If c_k is not unique, then go to step 6.

Step6: Determine the non-negative difference between the largest and the smallest values of both membership as well as non-membership values in each column and exhibit it as α_i, i=1, 2,…, n where n be the number of choice parameters.

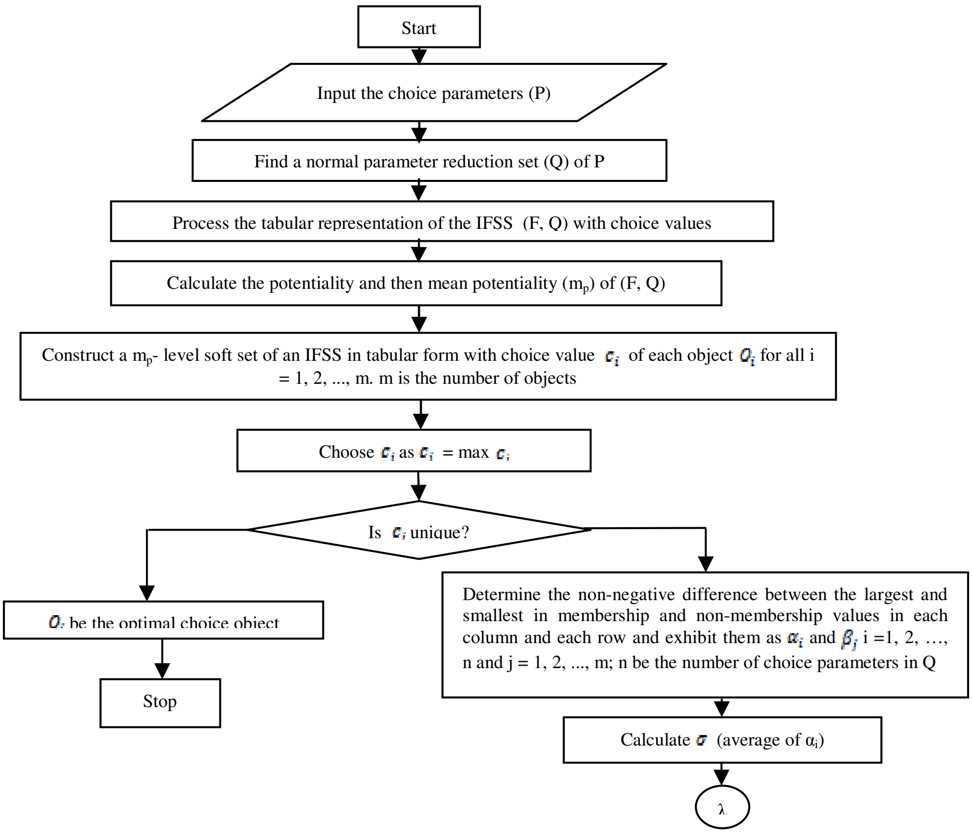
Step 7: The same procedure followed for each row (object) and denote the difference values as β_j where j=1, 2, …, m.

Step 8: Now take the average (α) of the α_i's upto ϱ significant figures and named it as σ.

Step 9: Then construct σ - level soft set and then compute the choice value c_i' for each object $O_i, \forall i$ =1, 2, … m from its tabular representation.

Step10: If c_i' is maximum and unique among $c_1', c_2', \ldots, c_m'$, where m is the number of objects (rows) then the optimal choice object is O_i' and then the process will be stopped. If c_i' is not unique, then go to step 11.

Step11: If l has more than one value then we have to consider the object corresponding to the mminimum value β_j in membership and non-membership value for j=1, 2, … m as the optimal choice of the decision maker (Figure 1).



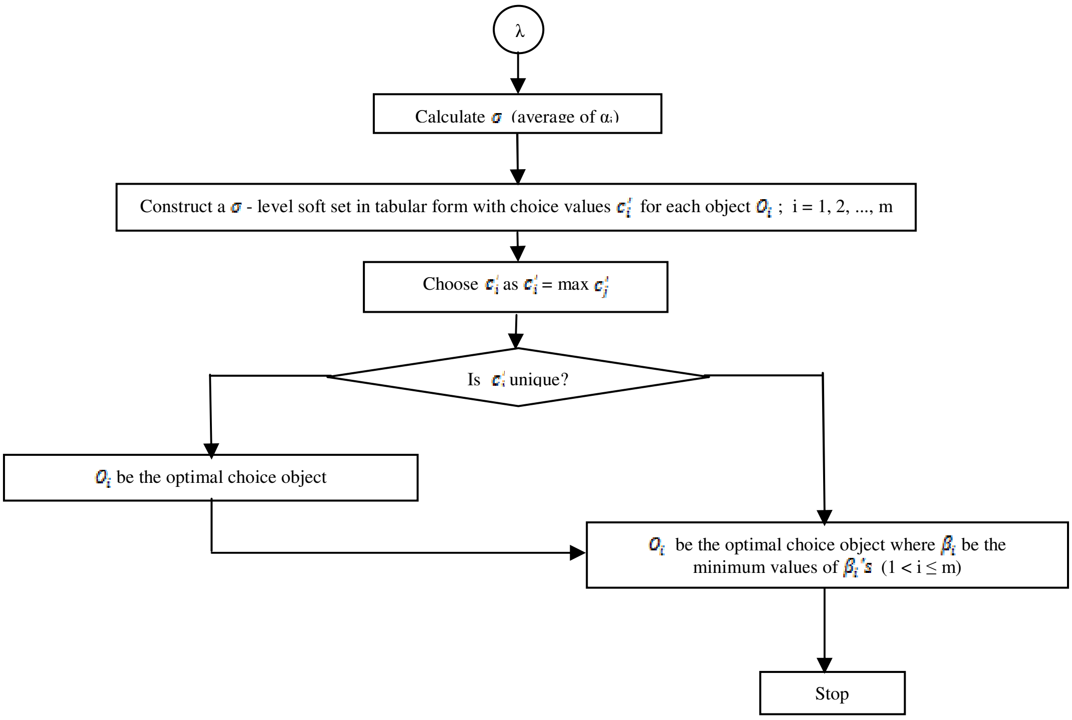


Fig 1: Flow chart of above synchronized algorithm

Example

To illustrate the basic idea of this algorithm, we apply it to some IFSSs based decision making problems. First, let us consider the decision making problem of example 3.3.1.

Step 1: Since P is indispensable, there does not exist any normal parameter reduction of P. Now the tabular representation of (F, P) with the choice values is given in Table 6.

Table 6: The tabular representation of (F, P)

	e_3	e_6	e_7	Choice value
h_1	(0.8, 0.1)	(0.7, 0.2)	(0.2, 0.6)	(1.7, 0.9)
h_2	(0.6, 0.2)	(0.9, 0)	(0.7, 0.2)	(2.2, 0.4)
h_3	(0.7, 0.2)	(0.5, 0.3)	(0.8, 0.2)	(2.0, 0.7)

Step 2: So the potentiality of (F, P) is, p_{ifs} = (5.9, 2.0).

Step 3: The mean potentiality of (F, P) is, $m_p = \left(\frac{5.9}{3\times3}, \frac{2.0}{3\times3}\right) = (0.6, 0.2)$. Since all the membership and non-membership values of the object concerned with this problem is one significant figure, therefore the maximum number of significant figures $\varrho = 1$. So m_p taken one significant figure.

Step 4: m_p- level soft set of (F, P) is given in Table 7.

Step 5: The optimal choice house is h_2, since $\text{Max}\{c_i, i = 1,2,3\} = c_2$.

Table 7: The tabular representation of L((F, P), 0.6,0.2)

	e_3	e_6	e_7	Choice value
h_1	1	1	0	2
h_2	1	1	1	3
h_3	1	0	1	2

4.1 Comparisons of the above two methods

Now calculate and compare the measure of performance of the example 3.3.1 and 4.3.1. Table 8 shows that the mean potentially approach method is better than Jiang's method. So, h_2 is an optimum house to buy for Mr. X.

Table 8: Comparison table

Name of the method	Solution of the problem	Measure of Performance
Jiang's method	Any one of the houses $h_1 or\ h_2\ or\ h_3$	(2.533,1.9) or (3.867,2.9) or (3.667,5.7)
Mean potentiality approach	h_2	(3.867,2.9)

Example

Consider a car classification problem. There are five new cars y_i (i = 1, 2, 3, 4, 5) to be classified in the Guangzhou car market in Guangdong, China, and six attributes: (1) *G*1: Fuel economy; (2) *G*2: Aerod. Degree; (3) *G*3: Price; (4) *G*4: Comfort; (5) *G*5: Design; and (6) *G*6: Safety, are taken into consideration in the classification problem. The characteristics of the ten new cars y_i (i = 1, 2, 3, 4, 5) under the six attributes G_j (j = 1, 2, 3, 4, 5, 6) are represented by the IFSSs, shown in Table 9.

Table 9: The tabular representation of (F, Q)

	G_1	G_2	G_3	G_4	G_5	G_6
y_1	(0.3,0.5)	(0.6,0.1)	(0.4,0.3)	(0.8,0.1)	(0.1,0.6)	(0.5,0.4)
y_2	(0.6,0.3)	(0.5,0.2)	(0.6,0.1)	(0.7,0.1)	(0.3,0.6)	(0.4,0.3)
y_3	(0.4,0.4)	(0.8,0.1)	(0.5,0.1)	(0.6,0.2)	(0.4,0.5)	(0.3,0.2)
y_4	(0.2,0.4)	(0.4,0.1)	(0.9,0)	(0.8,0.1)	(0.2,0.5)	(0.7,0.1)
y_5	(0.5,0.2)	(0.3,0.6)	(0.6,0.3)	(0.7,0.1)	(0.6,0.2)	(0.5,0.3)

By using Jiang's method:

Using mid level soft set method, the tabular representation of the corresponding mid level soft set of (F, P) with the choice values of the houses is given in Table 10. In this table, the choice values of the three houses are equal. In this situation, the decision maker may select any one of the houses (according to Jiang's method) as his optimal choice. So, mid -level soft set approach is not suitable for this problem.

Table 10: The tabular representation of L((F, Q), mid) with choice values

	G_1	G_2	G_3	G_4	G_5	G_6	Choice value
y_1	0	1	0	1	0	0	2
y_2	1	1	1	0	0	0	3
y_3	0	1	0	0	0	0	1
y_4	0	0	1	1	0	1	3
y_5	1	0	0	1	1	0	3

By using the Mean potentiality approach method:

Step 1: Since P is indispensable, there does not exist any normal parameter reduction of P. Now the tabular representation of (F, P) with the choice values is given in Table 11.

Table 11: The tabular representation of (F, Q) with choice values

	G_1	G_2	G_3	G_4	G_5	G_6	Choice value
y_1	(0.3,0.5)	(0.6,0.1)	(0.4,0.3)	(0.8,0.1)	(0.1,0.6)	(0.5,0.4)	(2.7,2.0)
y_2	(0.6,0.3)	(0.5,0.2)	(0.6,0.1)	(0.7,0.1)	(0.3,0.6)	(0.4,0.3)	(3.1,1.6)
y_3	(0.4,0.4)	(0.8,0.1)	(0.5,0.1)	(0.6,0.2)	(0.4,0.5)	(0.3,0.2)	(3.0,1.5)
y_4	(0.2,0.4)	(0.4,0.1)	(0.9,0)	(0.8,0.1)	(0.2,0.5)	(0.7,0.1)	(3.2,1.2)
y_5	(0.5,0.2)	(0.3,0.6)	(0.6,0.3)	(0.7,0.1)	(0.6,0.2)	(0.5,0.3)	(3.2,1.7)

Step 2: So the potentiality of (F, Q) is, p_{ifs} = (15.2, 8.0).

Step 3: The mean potentiality of (F, Q) is, $m_p = \left(\frac{15.2}{5\times6}, \frac{8.0}{5\times6}\right)$ = (0.5, 0.2).

Step 4: Now the tabular representation of the m_p level soft set of (F, Q) is given in the Table 12.

Step 5: Since some of the houses have the same choice value 3, so it is not unique. Hence, calculate the α_i and β_j values of (F, Q).

Step 6 and Step 7: The tabular representation of (F, Q) with α_i and β_j values are given in the Table 13.

Step 8: Now $\alpha = \frac{\sum_{i=1}^{6}\alpha_i}{6} = (0.4, 0.3)$. Therefore $\sigma = (0.4, 0.3)$.

Step 9: So, the tabular representation of the σ-level soft set of (F, Q) is given in the Table 14.

Step 10: Here $\max\{c_i; i = 1,2,3,4,5\} = 5 = \{c_2, c_5\}$. So we have to consider the β_j values for j = 2 and 5.

Step 11: Since β_2 *and* β_5 have the same. So we have to take either y_2 *or* y_5 as the optimal choice cars.

Table 12: The tabular representation of L((F, Q), 0.5,0.2) with choice values

	G_1	G_2	G_3	G_4	G_5	G_6	Choice value
y_1	0	1	0	1	0	0	2
y_2	0	1	1	1	0	0	3
y_3	0	1	1	1	0	0	3
y_4	0	0	1	1	0	1	3
y_5	1	0	0	1	1	0	3

Table 13: The tabular representation of (F, Q) with α_i and β_j values

	G_1	G_2	G_3	G_4	G_5	G_6	β_j
y_1	(0.3,0.5)	(0.6,0.1)	(0.4,0.3)	(0.8,0.1)	(0.1,0.6)	(0.5,0.4)	(0.7,0.5)
y_2	(0.6,0.3)	(0.5,0.2)	(0.6,0.1)	(0.7,0.1)	(0.3,0.6)	(0.4,0.3)	(0.4,0.5)
y_3	(0.4,0.4)	(0.8,0.1)	(0.5,0.1)	(0.6,0.2)	(0.4,0.5)	(0.3,0.2)	(0.5,0.4)
y_4	(0.2,0.4)	(0.4,0.1)	(0.9,0)	(0.8,0.1)	(0.2,0.5)	(0.7,0.1)	(0.7,0.5)
y_5	(0.5,0.2)	(0.3,0.6)	(0.6,0.3)	(0.7,0.1)	(0.6,0.2)	(0.5,0.3)	(0.4,0.5)
α_i	(0.4,0.3)	(0.5,0.5)	(0.5,0.3)	(0.2,0.1)	(0.5,0.4)	(0.4,0.3)	

Table 14: The tabular representation of L((F, Q), 0.4,0.3) with choice values

	G_1	G_2	G_3	G_4	G_5	G_6	Choice value
y_1	0	1	1	1	0	0	3
y_2	1	1	1	1	0	1	5
y_3	0	1	1	1	0	0	3
y_4	0	1	1	1	0	1	4
y_5	1	0	1	1	1	1	5

4.2 Comparisons of the above two methods

Using the above two methods to the example 4.3.3, we get the measure of performance results given in Table 15. This table shows that the mean potentially approach method is better than Jiang's method. So, y_5 is an optimum car.

Table 15: Comparison table

Name of the method	Solution of the problem	Measure of Performance
Jiang's method	Any one of the houses y_2 or y_4 or y_5	(3.470, 1.913) or (3.378, 1.494) or (3.617, 2.045)
Mean potentiality approach	y_2 or y_5	(3.470, 1.913) or (3.617, 2.045)

5 Parameter reduction procedure of an IFSSs

Parameter reduction is very important in the decision making problem. By this process the number of parameters in a problem can be efficiently minimized. So in a decision making problem, the parameter reduction helps us to present the key parameters. Here we are proposing the following algorithm to reduce the number of parameters in the set of choice parameters in an IFSS based decision making problem.

(i) At first apply the balanced algorithm to get the tabular representation (R) of the level soft set corresponding to the optimal choice object (O_{max}).

(ii) Secondly, apply the relational algebra based reduction algorithm which consists of the following steps:

(a) First construct a subset E_1 of the choice parameter set P such that,

$$E_1 = \{e : e \in E \text{ and } (O_{max}) \in F(e)\}$$

(b) Then compute R_{E_1} which is the result of a projection operation on E_1 of R, i.e., R_{E_1} is a relation comprising after selecting the columns corresponding to all parameters of E_1 from R.

(c) Compute each R_{e_i} $\forall e_i \in E_1$, where R_{e_i} is a relation obtained by selection operation on R which satisfies the selecting condition $e_i = 0$; i = 1, 2, . . . , p, where p be the cardinality of the set E1.

(d) Finally, apply union operation on the relations R_{e_i}, $e_i \in E_1$ in such a way that the difference operation of the resulting relations from the relation R_{E_1}, i.e., the combinations $R_{E_1} - \bigcup_k R_{e_i}$ gives a relation comprising a row corresponding the optimal choice object O_{max}, where $\bigcup_k R_{e_i}$ is a relation resulting from union set of k - number of R_{e_i} (1 $\leq$ k $\leq$ p). Hence the set $\{e_i : \cup_k R_{e_i}\}$ be the reduced parameter set of P (Figure 2).

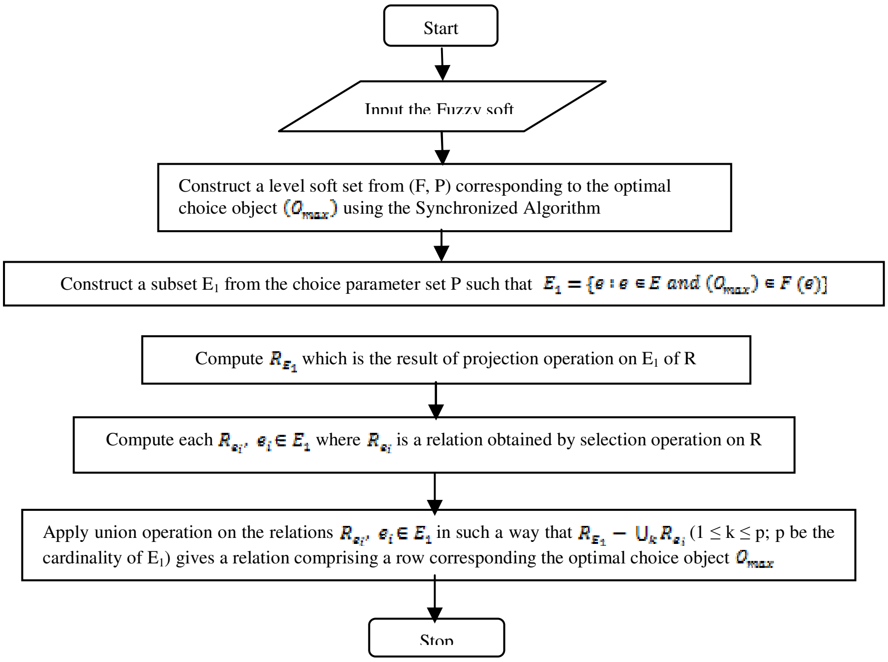


Fig. 2: Flowchart for parameter reduction procedure

6 The diagnosis of a disease from the myriad of symptoms

In this section, we will illustrate the application of the above two algorithms in the medical diagnosis problem. The diagnostic process is the method by which health professionals select one disease over another, identifying one as the most likely cause of a person's symptoms. Reaching an accurate conclusion depends on the timing and the sequence of the symptoms, past medical history and risk factors for certain diseases, and a recent exposure to disease. The physician, in making a diagnosis, also relies on various other clues such as physical signs, nonverbal signals of distress, and the results of selected laboratory and radiological and other imaging tests.

Example

Now we consider from medical science seven symptoms such as abdominal pain, fever, headache, weight loss, muscle pain, nausea vomiting, diarrhea which have more or less contribution in four diseases such as typhoid, peptic ulcer, food poisoning, acute viral hepatitis. Now from medical statistics, the degree of availability and non- availability of these seven symptoms in these four diseases are observed as follows:

Let the finite universe, $U = \{d_1, d_2, d_3, d_4\}$ = {typhoid, peptic ulcer, food poisoning, acute viral hepatitis} be the four diseases and the set of parameters E = {abdominal pain, fever, headache, weight loss, muscle pain, nausea vomiting, diarrhea} be the seven symptoms. Then the IFSS (F, E) is defined as,

$$(F, E) = \{ \text{ Abdominal pain } (e_1) = \left\{ \frac{d1}{(0.3,0.6)}, \frac{d2}{(0.9,0)}, \frac{d3}{(0.6,0.3)}, \frac{d4}{(0.2,0.7)} \right\},$$

$$\text{Fever } (e_2) = \left\{ \frac{d1}{(0.8,0.1)}, \frac{d2}{(0.2\,,0.8)}, \frac{d3}{(0.3,0.5)}, \frac{d4}{(0.6,0.2)} \right\},$$

$$\text{Headache } (e_3) = \left\{ \frac{d1}{(0.2,0.6)}, \frac{d2}{(0.1,0.8)}, \frac{d3}{(0.1,0.9)}, \frac{d4}{(0.4,0.5)} \right\},$$

$$\text{Weight loss } (e_4) = \left\{ \frac{d1}{(0.2,0.7)}, \frac{d2}{(0.1,0.8)}, \frac{d3}{(0.6,0.3)}, \frac{d4}{(0.2,0.8)} \right\},$$

$$\text{Muscle pain } (e_5) = \left\{ \frac{d1}{(0.2,0.6)}, \frac{d2}{(0.1,0.9)}, \frac{d3}{(0.2,0.6)}, \frac{d4}{(0.2,0.7)} \right\},$$

$$\text{Nausea vomiting } (e_6) = \left\{ \frac{d1}{(0.1,0.8)}, \frac{d2}{(0.1,0.7)}, \frac{d3}{(0.6,0.2)}, \frac{d4}{(0.6,0.3)} \right\},$$

$$\text{Diarrhea } (e_7) = \left\{ \frac{d1}{(0.2,0.6)}, \frac{d2}{(0.1,0.7)}, \frac{d3}{(0.7,0.2)}, \frac{d4}{(0.1,0.9)} \right\} \}$$

Suppose a patient who is suffering from a disease have the symptoms P(abdominal pain(e_1), fever(e_2), headache(e_3), nausea vomiting(e_6) and diarrhea(e_7)). Now the problem is how the physician detects the actual disease with effective symptoms among these four diseases for that patient. To solve this problem, first we detect the disease which is most suited with the observed symptoms of the patient and then secondly we find the actual symptoms which are optimal for that disease. The tabular representation of (F, P) with choice values are given in the table 16.

Table 16: The tabular representation of L(F, P) with choice values

	e_1	e_2	e_3	e_6	e_7	Choice value
d_1	(0.3, 0.6)	(0.8, 0.1)	(0.2, 0.6)	(0.1, 0.9)	(0.2, 0.6)	(1.6, 2.8)
d_2	(0.9, 0)	(0.2, 0.8)	(0.1, 0.8)	(0.1, 0.7)	(0.1, 0.7)	(1.4, 3)
d_3	(0.6, 0.3)	(0.3, 0.5)	(0.1, 0.9)	(0.6, 0.2)	(0.7, 0.2)	(2.3, 2.1)
d_4	(0.2, 0.7)	(0.6, 0.2)	(0.4, 0.5)	(0.6, 0.2)	(0.1, 0.9)	(1.9, 2.5)

By using the Mean potentiality approach:

Since P is indispensable, there does not exist any normal parameter reduction of P. Hence the mean potentiality of (F, P) is, $m_p = (0.3, 0.5)$. The tabular representation of the m_p –level soft set of (F, P) is given in the table 17.

Here the choice value of the disease $d_3$is maximum among the other four diseases and therefore the disease most suitable with the symptoms is food poisoning d_3.

Table 17: The tabular representation of L ((F, P); (0.3, 0.5)) with choice values

	e_1	e_2	e_3	e_6	e_7	Choice value
d_1	0	1	0	0	0	1
d_2	1	0	0	0	0	1
d_3	1	1	0	1	1	4
d_4	0	1	1	1	0	3

By using Jiang's method:

Using a mid level soft set (Table 18): Now from this table we observe that either d_3 *or* d_4 be the optimal choice disease corresponding to the maximum choice value 3.

Table 18: Tabular representation of L ((F, P); mid) with choice value

	e_1	e_2	e_3	e_6	e_7	Choice value
d_1	0	1	1	0	0	2
d_2	1	0	0	0	0	1
d_3	1	0	0	1	1	3
d_4	0	1	1	1	0	3

Using topbottom level soft set (Table 19): Again from this table we observe that either d_3 *or* d_4 be the optimal choice disease corresponding to the maximum choice value 2.

Table 19: Tabular representation of L ((F, P); topbottom) with choice value

	e_1	e_2	e_3	e_6	e_7	Choice value
d_1	0	1	0	0	0	1
d_2	1	0	0	0	0	1
d_3	0	0	0	1	1	2
d_4	0	0	1	1	0	2

Hence, according to the Jiang's method the physician may detect that the patient is suffering from either food poisoning d_3 or acute viral hepatitis d_4 .

Now the effective symptoms of this disease will be found out according to the proposed algorithm of parameter reduction.

Let d_3 will be denoted by d_{max}. Now construct a subset E_1 of the choice parameter set P such that

$$E_1 = \{e: e \in E \text{ and } d_{max} \in F(e)\}$$

Therefore, in this case $E_1 = \{e_1, e_2, e_6, e_7\}$. Now let the relation, i.e., the tabular representation of the m_p-level soft set of (F, P) be denoted by R. Then compute R_{E_1} which is the result of a projection operation on E_1 of R, i.e. R_{E_1}, is a relation comprising after selecting the column corresponding to e_1, e_2, e_6, e_7 from R. So, the tabular representation of the relation R_{E_1} is given in the table 20.

Table 20: The tabular representation of the relation R_{E_1}

	e_1	e_2	e_6	e_7
d_1	0	1	0	1
d_2	1	0	0	0
d_3	1	1	1	1
d_4	0	1	1	0

Now applying the selection operation from [28], on the relation R_{E_1} to get the other four new relations R_{e_i}; i=1,2,6,7 such that if either the membership value or non-membership value or both the values are zero, we select the corresponding disease, i.e., $e_1 = 0$, $e_2 = 0$, $e_6 = 0$ and $e_7 = 0$ respectively in the table R_{E_1}.

$$R_{e_i} = \text{select * from } R_{E_1} \text{ where } e_i = 0 : \ i = 1, 2, 6 \text{ and } 7.$$

Using these selections, we get the relations given in the tables 21-24.

Table 21: The tabular representation of the relation R_{e_1}

	e_1	e_2	e_6	e_7
d_1	0	1	0	1
d_4	0	1	1	0

Table 22: The tabular representation of the relation R_{e_2}

	e_1	e_2	e_6	e_7
d_2	1	0	0	0

Table 23: The tabular representation of the relation R_{e_6}

	e_1	e_2	e_6	e_7
d_1	0	1	0	1
d_2	1	0	0	0

Table 24: The tabular representation of the relation R_{e_7}

	e_1	e_2	e_6	e_7
d_1	0	1	0	1
d_2	1	0	0	0
d_4	0	1	1	0

Finally, we apply union operation on the relations $R_{e_1}, R_{e_2}, R_{e_6}, R_{e_7}$ in such a way that the difference operation of the resulting relations from the relation R_{E_1} gives the following relation comprising a row corresponding the optimal choice object d_{max}.

Now this relation can be obtained by each of the ways $\left\{R_{E_1} - \{R_{e_1} \cup R_{e_2}\}\right\}$, $\left\{R_{E_1} - \{R_{e_1} \cup R_{e_6}\}\right\}$, $\left\{R_{E_1} - \{R_{e_1} \cup R_{e_7}\}\right\}$, $\left\{R_{E_1} - \{R_{e_2} \cup R_{e_7}\}\right\}$, $\left\{R_{E_1} - \{R_{e_6} \cup R_{e_7}\}\right\}$, $\{R_{E_1} - R_{e_7}\}$.

Therefore $\{R_{e_1} \cup R_{e_2}\}$, $\{R_{e_1} \cup R_{e_6}\}$, $\{R_{e_1} \cup R_{e_7}\}$, $\{R_{e_2} \cup R_{e_7}\}$, $\{R_{e_6} \cup R_{e_7}\}$, $\{R_{e_7}\}$ are the parameter reductions of (F, P). This result indicates that when the doctor identifies the most suitable disease with the symptoms (P) of the patient, then the symptoms list (P) can be reduced into either $\{e_1, e_2\}$ or $\{e_1, e_6\}$ or $\{e_1, e_7\}$ or $\{e_2, e_7\}$ or $\{e_6, e_7\}$ or $\{e_7\}$ are the effective symptoms of the patient. i.e., {abdominal pain, fever} or {abdominal pain, nausea vomiting} or {abdominal pain, diarrhea} or {fever, diarrhea} or {nausea vomiting, diarrhea} or {diarrhea} which confirm the diagnosis of the patient suffering from food poisoning (d_3).

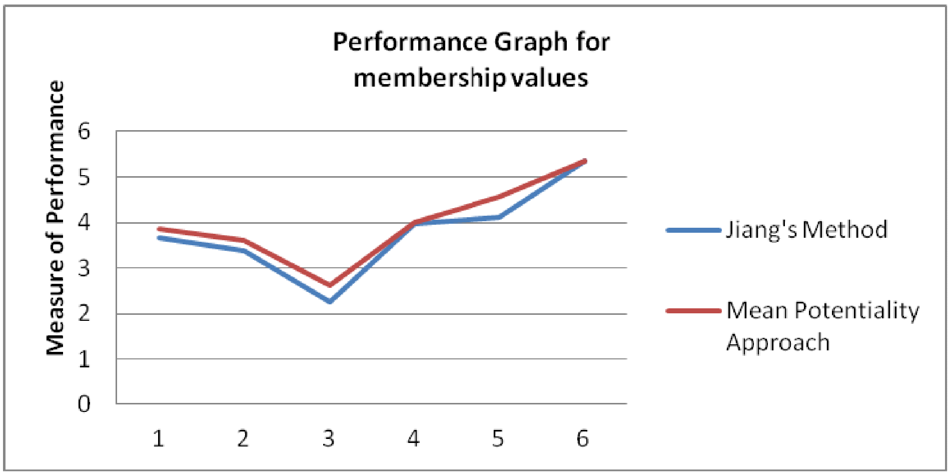
6.2 Comparisons of the above two methods

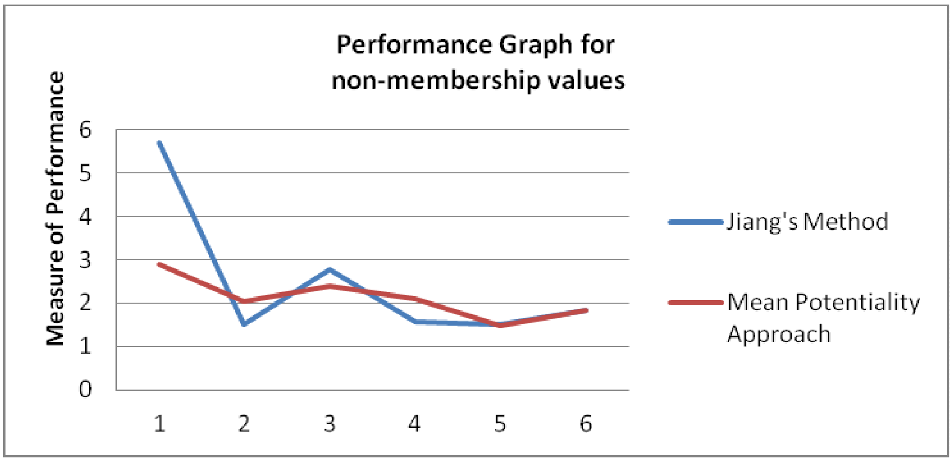
Using the above two methods, we get the measure of performance results given in Table 25. This table shows that the mean potentially approach method is better than Jiang's method.

Table 25: Comparison table

Name of the method	Solution of the problem	Measure of Performance
Mean potentiality approach	d_3	(2.63, 2.394)
Jiang's method	Any one of the houses d_3 or d_4	(2.63, 2.394) or (2.25, 2.763)

In addition, other three data sets from [32] are considered and have compared with the mean potentiality approach. The following graphs represent the measure of performance of membership and non-membership vales of these two methods respectively, by which it can be clearly understood that this mean potentiality approach is more efficient and suitable for decision making problems.





7 Weighted IFSSs based decision making

In this section we will present an adjustable approach to weighted IFSSs based decision making problems by extending the approach to weighted fuzzy soft sets based decision making.

By definition, every IFSSs can be considered as weighted IFSSs. Similarly to weighted fuzzy soft sets, the notion of weighted IFSSs provides a mathematical framework for modeling and analyzing the decision making problems in which all the choice parameters may not be of equal importance. These differences between the importances of parameters are characterized by the weight function in a weighted IFSS. Now we show the adjustable approach to weighted IFSSs based decision making by using level soft sets.

In the above example 6.1, suppose that the physician has imposed the following weights for the symptoms in P: for the symptom ‘‘abdominal pain”, $w_1 = 0.7$; for the symptom ‘‘fever”, $w_2 = 0.8$; for the symptom ‘‘headache”, $w_3 = 0.6$; for the symptom ‘‘nausea vomiting”, $w_4 = 0.9$; and for the symptom ‘‘diarrhea”, $w_5 = 0.9$. Thus we have a weight function $\omega : A \to [0,1]$ and the IFSS $\bar{\omega} = (F, A)$ in this example is changed into a weighted IFSS $\xi = (F, A, \omega)$ with its tabular representation as shown in Table 26.

Table 26: The tabular representation of L ((F, P); (0.3, 0.5)) with weighted choice values

	e_1, $w_1 = 0.7$	e_2, $w_2 = 0.8$	e_3, $w_3 = 0.6$	e_6, $w_4 = 0.9$	e_7, $w_5 = 0.9$	Weighted choice value
d_1	0	1	0	0	0	0.8
d_2	1	0	0	0	0	0.7
d_3	1	1	0	1	1	3.3
d_4	0	1	1	1	0	2.3

From Table 26, it follows that the maximum weighted choice value is 3.3 and so the disease d_3 is maximum among the other four diseases and therefore the disease most suitable with the symptoms is food poisoning d_3.

8 Conclusion and Future work

In this paper, a Mean Potentiality Approach for a balanced solution of an IFSS based decision making problem is proposed using level soft sets. The proposed procedure was applied to a decision making problem discussed in Jiang’s [29] and it is shown that the proposed procedure overcomes the limitation of the method in [29]. Further, we applied parameter reducing algorithm to identify the key parameters which are helping to make decisions. We believe that these theories have a lot of future and may serve to solve many decision making problems.

As far as future directions are concerned, these will include extending soft rough set to intuitionistic fuzzy case and studying the adjustable approach to soft rough set based IFSS based decision making. It is also desirable to further apply level soft sets of IFSSs to other practical applications based on IFSSs.

ACKNOWLEDGEMENTS

The authors are grateful to the referees for their valuable suggestions in rewriting the paper in the present form. The authors are also thankful to the reviewers for their valuable comments which helped us to improve the paper.

References

[1] A.R. Roy, P.K. Maji, "A fuzzy soft set theoretic approach to decision making problems", J. Comput. Appl. Math., Vol. 203, No.(2), (2007), pp. 412–418.

[2] D. Chen, E.C.C. Tsang, D.S. Yeung, X. Wang, "The parameterization reduction of soft sets and its applications", Comput. Math. Appl.,Vol.49, No.(5–6), (2005), pp.757–763.

[3] D. Molodtsov, "Soft set theory – first results", Comput. Math. Appl., Vol.37, No.(4–5), (1999), pp. 19–31.

[4] F. Feng, C. Li, B. Davvaz, M.I. Ali, "Soft sets combined with fuzzy sets and rough sets: a tentative approach", Soft Comput., Vol.14, No.(9), (2010), pp. 899–911.

[5] F. Feng, Y.B. Jun, X. Liu, L. Li, "An adjustable approach to fuzzy soft set based decision making", J. Comput. Appl. Math., Vol.234, No.(1), (2010), pp. 10–20.

[6] F. Feng, Y.B. Jun, X. Zhao, "Soft semi rings", Comput. Math. Appl., Vol. 56, No. (10), (2008), pp. 2621–2628.

[7] Feng Feng, "Soft rough sets applied to multicriteria group decision making", Annals of fuzzy mathematics and informatics, Vol.2, No. (1), (2011), pp. 69-80.

[8] Feng Feng, Y. Li, Violeta Leoreanu-Fotea, "Application of level soft sets in decision making based on interval valued fuzzy soft sets", Computers and Mathematics with Applications, Vol. 60, (2010), pp. 1756-1767.

[9] H. Aktas, N. Cagman, "Soft sets and soft groups", Inform. Sci., Vol.177, No. (13), (2007), pp. 2726–2735.

[10] K. Atanassov, "Intuitionistic fuzzy sets", Fuzzy Sets Syst., Vol.20, No.(1) (1986), pp. 87–96.

[11] M.I. Ali, F. Feng, X. Liu, W.K. Min, M. Shabir, "On some new operations in soft set theory", Comput. Math. Appl., Vol. 57, No. (9), (2009), pp. 1547–1553.

[12] N. Cagman, S. Enginoglu, "Soft matrix theory and its decision making", Comput. Math. Appl., Vol. 59, No.(10), (2010), pp. 3308–3314.

[13] N. Cagman, S. Enginoglu, "Soft set theory and uni–int decision making", Eur. J. Oper. Res., Vol. 207, No.(2), (2010), pp. 848–855.

[14] P. Majumdar, S.K. Samanta, "Generalized fuzzy soft sets", Comput. Math. Appl., Vol. 59, No. (4), (2010), pp. 1425–1432.

[15] P. Majumdar, S.K. Samanta, "A Generalised Fuzzy Soft Set Based Student Ranking System", Int. J. Advance. Soft Comput. Appl., Vol. 3, No. 3, (2011) pp. 1-10.

[16] P.K. Maji, A.R. Roy, R. Biswas, "An application of soft sets in a decision making problem", Comput. Math. Appl., Vol.44, No. (8–9), (2002), pp. 1077–1083.

[17] P.K. Maji, A.R. Roy, R. Biswas, "On intuitionistic fuzzy soft sets", J. Fuzzy Math., Vol.12, No. (3), (2004), pp. 669–683.

[18] P.K. Maji, R. Biswas, A.R. Roy, "Soft set theory", Comput. Math. Appl., Vol. 45, No.(4–5), (2003), pp. 555–562.

[19] P.K. Maji, R. Biswas, A.R. Roy, "Fuzzy soft sets", J. Fuzzy Math., Vol.9, No. (3), (2001), pp. 589–602.

[20] P.K. Maji, R. Biswas, A.R. Roy, "Intuitionistic fuzzy soft sets", J. Fuzzy Math., Vol.9, No.(3), (2001) ,677–692.

[21] T. Mitra Basu, Nirmal Kumar Mahapatra, Shyamal Kumar Mondal, "A balanced solution of a fuzzy soft set based decision making problem in medical science", Appl. Soft Comp., Vol.12, No.(2012), pp. 3260-3275.

[22] U. Acar, F. Koyuncu, B. Tanay, "Soft sets and soft rings", Comput. Math. Appl., Vol.59, No.(11), (2010), pp. 3458–3463.

[23] W. Xu, J. Ma, S. Wang, G. Hao, "Vague soft sets and their properties", Comput. Math. Appl., Vol.59, No.(2), (2010), pp. 787–794.

[24] X.B. Yang, T.Y. Lin, J.Y. Yang, Y. Li, D. Yu, "Combination of interval-valued fuzzy set and soft set", Comput. Math. Appl., Vol.58 , No.(3),(2009), pp. 521–527.

[25] Y. Jiang, Y. Tang, Q. Chen, H. Liu, J. Tang, "Interval-valued intuitionistic fuzzy soft sets and their properties", Comput. Math. Appl., Vol.60, No.(3),(2010), pp. 906–918.

[26] Y. Zou, Z. Xiao, "Data analysis approaches of soft sets under incomplete information", Knowledge-Based Syst., Vol.21, No.(8), (2008), pp. 941–945.

[27] Y.B. Jun, K.J. Lee, A. Khan, “Soft ordered semi groups”, Math. Logic Quart., Vol.56, No.(1), (2010), pp. 42–50.

[28] Y.B. Jun, “Soft BCK/BCI-algebras”, Comput. Math. Appl., Vol.56, No.(5), (2008), pp. 1408–1413.

[29] Y. Jiang, Yong Tang, Qimai Chen, “An adjustable approach to intuitionistic fuzzy soft sets based decision making”, Appl. Math. Model., Vol.35,(2011), pp. 824-836.

[30] Z. Kong, L. Gao, L. Wang, Comment on ‘‘A fuzzy soft set theoretic approach to decision making problems”, J. Comput. Appl. Math., Vol.223, No.(2), (2009), pp.540–542.

[31] Z. Kong, L. Gao, L. Wang, S. Li, “The normal parameter reduction of soft sets and its algorithm”, Comput. Math. Appl., Vol.56, No.(12), (2008), pp. 3029-3037.

[32] Zeshui Xu, Junjie Wu, “Intuitionistic fuzzy C-means clustering algorithms”, Journal of Systems Engineering and Electronics Vol. 21, No. 4, (2010), pp. 580–590.